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DESIGN OF QUIET PERMANENT MAGNET SYNCHRONOUS ELECTRICAL MOTORS BY OPTIMUM SKEW ANGLE

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This paper presents the phenomenon of electromagnetic noise and vibrations in electrical machines, and focuses on the analysis of the noise reduction of skewing in surface permanent magnet synchronous machines (SPMSM). The origin of the main rotating and pulsating magnetic forces is detailed in terms of stator slotting and rotor magnet harmonics, and the optimal value of stator and rotor skew is derived. Some numerical electromagnetic and vibroacoustic simulations are then carried and confirm the analytical work. It is shown that the minimization of cogging torque does not necessarily lead to lower acoustic noise. Depending on the vibration waves responsible for noise (pulsating or rotating), the optimal rotor skew value is different. It is therefore necessary to run electromagnetic and vibroacoustic simulations to optimize the noise level of the electrical machine.

1. Noise and vibrations in electrical machines

The vibro-acoustic design of electrical machines is as important as its electromagnetic or thermal design in an increasing number of applications. This is the case of rotating machines working close to human presence in industrial applications (e.g. pumps, reciprocating compressors), household appliances (e.g. HVAC, electric curtains), transportation (e.g. ships, trains, electric cars, pedal-assist bicycles) and energy sectors (e.g. wind turbine generators). More generally, the vibration and acoustic noise levels of electrical machines have to be controlled and reduced as part of their global environmental impact.

Noise and vibration sources of electrical machines are usually classified into mechanical, aerodynamic and magnetic sources [1]. Mechanical noise and vibrations can come from bearings, gears and brush commutators, whereas aerodynamic noise is due to air pressure periodic variations coming for instance from mounted fans or air-gap vortices due to slotting effects ; in high speed machines, these aerodynamic forces can also excite the structural modes of the machine and give high pitch noise [2]. The transfer path of these exciting forces are illustrated in Figure 1.

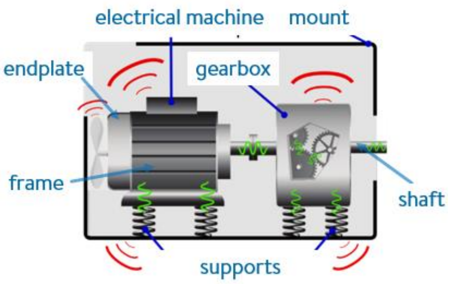

Figure 1: Main vibroacoustic transfer paths of an electrical machine (green: vibration, red: noise).

Magnetic vibration and audible noise are due to the deflections of the magnetic circuits (stator and rotor) of the electrical machine under magnetic forces; magnetic forces are defined as forces arising from the presence of a magnetic field, which can be due to permanent sources as in permanent magnet synchronous machines, or induced by some current sources as in induction machines. Two types of magnetic forces occur in electrical machines: magnetostrictive and Maxwell force [3]. Qualitatively, magnetostrictive forces occur in magnetic sheets and tend to shrink the material along the field lines, whereas Maxwell forces are mainly located at the lamination interface with air and globally tends to bring the stator closer to the rotor (law of minimal reluctance). These forces are illustrated in Figure 2.

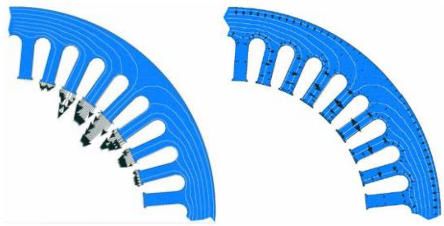

Figure 2: Representation of Maxwell forces (left) and magnetostriction forces (right) on the stator of an induction motor.

Both phenomena are quadratic function of the flux density and can result in radial deflection of the yoke and acoustic noise. Although the scientific debate is still not closed on the respective contribution of these forces in the different electrical machine topologies and power ranges [4-7], this paper only focuses on Maxwell forces.

2. Electromagnetically-induced acoustic noise

Figure 2 shows that Maxwell forces are not uniform along the airgap of the electrical machines: they vary both in time and along the airgap angle. They can therefore be developed as a series of rotating radial and tangential force waves with a given frequency f and wavenumber r. $r=0$ corresponds to the average radial magnetic force, or torque in tangential direction; $r=1$ corresponds to what is called unbalanced magnetic pull; other circumferential wavenumbers $r>1$ represents elliptical force waves (see Figure 3). These forces apply to both stator and rotor.

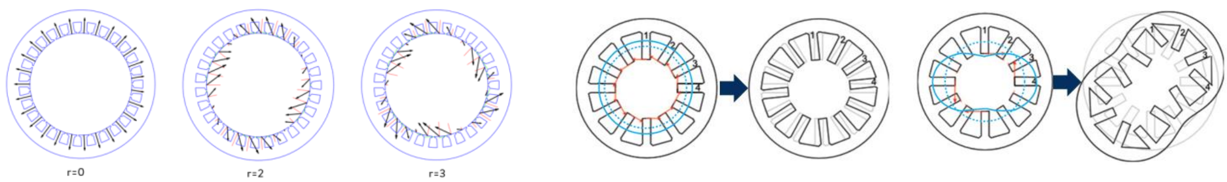

Figure 3: At the left: Examples of Maxwell force waves (radial and tangential components) applied on a stator (output from MANATEE software [12]). At the right: Effect of tangential magnetic force waves (left: r=0; right: r=2) [11].

Radial force wave $r=0$ creates a pulsating deflection of the stator where all the points move in phase (“breathing mode”). This force is significantly present in electrical machines, especially permanent magnet machines used for automotive electrical traction [8]. Radial force wave $r=0$ has no impact on a plain inner rotor which is too stiff, but it can create significant deflection of an outer rotor [9]. Tangential force wave $r=0$ creates a torsion on both stator and rotor. Stator pure torsional deflections cannot radiate efficiently acoustic noise, unless it generates torsion of the frame with radial fins, or it has asymmetrical boundary conditions [10]. Torsional deflections of the rotor, also named torque ripple or cogging torque at no-load for permanent magnet synchronous machines, can lead to structure-borne acoustic noise when propagating to bearings, end-plates and gearbox (Figure 1). Radial and tangential force wave r>1 creates radial deflection of the stator. Tangential forces are often neglected by some authors assuming they create only tangential deflections of the yoke, but tangential forces applied to stator tooth tips create radial deflection of the yoke through the bending moment of teeth (see Figure **3**). However, this effect cannot occur for outer rotor PM machines where tangential forces create torsional deflections only. Similar to *r=0*, tangential and radial force wave with *r>1* cannot create significant deflection of a plain inner rotor due to its high stiffness.

The stator structural modes can be modelled as cylindrical modes characterized by (m,n), where *m* is the circumferential order of the modal shape and n the longitudinal one, and fmn the associated natural frequency (Figure 4). Maxwell force waves can enter in resonance with a structural mode of the stator lamination when matching the corresponding natural frequency (f=fm0) and modal shape (r=m) [13]. Stator pure circumferential modes (m,0) can be excited by tangential and radial force waves of wavenumber r=m. The rotor shaft bending mode can be excited by magnetic force wave of wavenumber *r=1*.

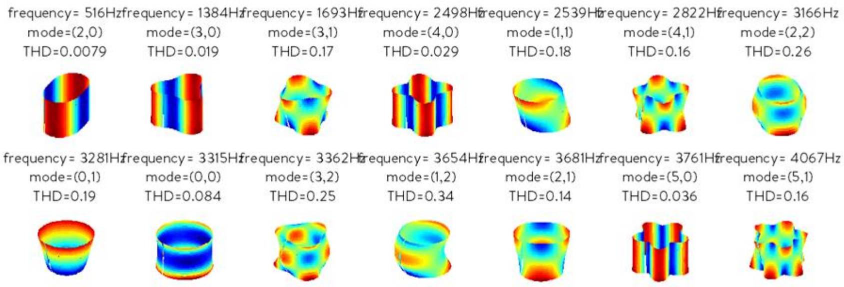

Figure 4: Example of a cylinder equivalent modal basis of a stator (output from MANATEE software [12]) in clamped-free boundary conditions.

Electromagnetically-induced acoustic noise can be a key contributor to the global sound power level of electrical machines for several reasons. Firstly, due to the strong harmonic nature of magnetic forces in both time and space domains, magnetic noise is characterized by strong tonalities (emergence of a given harmonic above the background noise) and usually occur in the most sensitive frequencies of the human ear (1 to 10 kHz). This makes it sound particularly unpleasant, especially as some progress is made on the reduction of the other broad-band noise sources (e.g. use of water cooling technology instead of fans). Secondly, the design optimization of electrical machines with respect to cost, weight and efficiency tend to minimize materials, leading to thinner yoke width and increased vibration levels. Finally, in electrical transportation, electrical machines are run at variable speed and the magnetic force harmonics sweep a wide frequency range, increasing the number of resonances with stator and rotor modes. Besides that, the variable frequency drive enriches the harmonic content of magnetic forces. The level of electromagnetically-induced acoustic noise is therefore increased.

3. Skewing noise reduction technique

3.1 Principle

A well-known technique to reduce the magnetic noise and vibration of electrical machines is named skewing. It consists of varying the position of slots or magnets along the axial direction of the machine, filtering out some of the magnetic force harmonics due to stator to rotor magnetic interactions. Usually it is done by a linear inclination of slots and poles, but some authors have investigated more complex shapes [14-15]. The skewing of the wound stator is more complicated to implement, so the rotor is more often skewed. Permanent magnets are generally manufactured in bars, so that a continuous angular shift of magnetic poles is not possible and an equivalent stepped-skew technique is used [16].

3.2 Force harmonics in PMSM

3.2.1 Principle

In this part the origin of some harmonic forces in PMSM are analytically derived. This is done by simply noting *{f, r}* a progressive wave *A* travelling in the airgap with frequency *f* and wavenumber *r*:

$$A(t,\alpha) = A\cos(2\pi ft - r\alpha + \phi) \equiv \{f, r\}. \quad (1)$$

This way the product of two waves is $\{f_1, r_1\}\{f_2, r_2\}=\{f_1+/- f_2, r_1+/-r_2\}$

The magnetic force waves are quadratic function of the airgap flux density, which is the sum of stator and rotor induced flux in the linear case. Stator and rotor airgap flux can be itself decomposed as the product of the airgap permeance per unit area *P* and magnetomotive force (mmf) *F*. The stator permeance introduces slotting effects:

$$P \equiv \{0, k_s Z_s\}. \quad (2)$$

where k_s is an integer and Z_s is the number of stator slots. The larger is k_s, the lower is the magnitude of the permeance harmonic. The rotor magnetomotive force due to North and South pole alternance can be written as:

$$F_r \equiv \{(2h_r+1) f_s, (2h_r+1)p\} = \{\nu_r f_R, \nu_r\}. \quad (3)$$

where f_s is the fundamental supply frequency, f_R the mechanical frequency and *p* the number of pole pairs. Again the larger h_r is, the lower is the magnitude of the mmf harmonic. For distributed windings, the stator magnetomotive force waves can be written as [13]:

$$F_s \equiv \{f_s, (2q_s h_s+1)p\}. \quad (4)$$

where q_s is the number of phases. The larger h_s is, the lower is the magnitude of the mmf harmonic.

3.2.2 Case of zero-th wavenumber (r=0)

Magnetic forces due to wavenumber $r=0$ are very special. Their time-averaged tangential component corresponds to the average electromagnetic torque produced by the machine, which is generally maximized by the designer, and the associated time harmonics are torque ripple components which are generally minimized by the designer. Their time-averaged radial component corresponds to a static magnetic stress and has only a potential effect on the shift of structural modes natural frequencies, whereas the associated time harmonics are a potential source of parasitic vibration and noise. Besides that, in PMSM, the magnitudes of radial and tangential force harmonic of same frequency and wavenumber are correlated [17]. One source of pulsating harmonic forces is the interactions between slots and magnet harmonics defined by

$$0 = (2h_{r2}+1)p \pm (2h_{r1}+1)p \pm k_s Z_s. \quad (5)$$

One can see that the sum and difference of the two rotor mmf harmonics are multiples of Zs and 2p, therefore it is a common multiple of Zs and 2p. If $N_c=LCM(Z_s,2p)$ is noted as the least common multiple between the slot number and pole number, the sum and difference of the rotor mmf harmonics giving rise to $r=0$ forces are thereof multiple of N_c and can be written as nN_c where *n* is an integer. The larger *n* is, the larger is the associated slotting harmonic and the lower are the pulsating forces. The largest magnitude pulsating force is therefore given by rotor mmf harmonic combination that are exactly equal to N_c:

$$N_c = LCM(Z_s, 2p) = (2h_{r2}+1)p \pm (2h_{r1}+1)p. \quad (6)$$

This equation is equivalent to

$$N_c/(2p) - 1 = h_{r1} + h_{r2} \,. \quad (7)$$

$$N_c/(2p) = h_{r1} - h_{r2} \,. \quad (8)$$

For the summation case (+), when h_{r1} goes from 0 to $N_c/(2p)-1$ the first rotor mmf harmonic goes from p to N_c-p (N_c >=2p) and the second mmf harmonic from N_c-p to p.

For the difference case (-), when h_{r1} goes from $N_c/(2p)$ to the infinity the first rotor mmf harmonic goes from N_c+p to the infinity and the second mmf harmonic from p to the infinity.

The force wave is proportional to the product of the rotor mmf. In the first case its wavenumber is given by:

$$\nu_{r1}\nu_{r2} = \nu_{r1}(N_c - \nu_{r1}) = p(2h_{r1}+1)\,p(2(N_c/(2p) - 1 - h_{r1}) + 1) \,. \quad (9)$$

In the second case it is given by

$$\nu_{r1}\nu_{r2} = \nu_{r1}(\nu_{r1} - N_c) = p(2h_{r1}+1)\,p(2(h_{r1} - N_c/(2p)) + 1) \,. \quad (10)$$

Both are quadratic functions of the mmf rank h_{r1} and looking for their maximum magnitude gives in the first case the mmf harmonics (p, N_c-p) and in the second case the harmonics (N_c+p,p). This shows that the rotor mmf harmonics given by $\nu_r = N_c \pm p$ are responsible for the largest magnetic pulsating forces (radial and tangential) in PMSM. This can be generalized to give the family of rotor magnet mmf harmonics involved in the pulsating forces ($r=0$) at frequency $f=nN_cf_R$:

$$\nu_r = nN_c \pm p \,. \quad (11)$$

where n is a positive integer. The corresponding stator slotting harmonics are simply given by $k_sZ_s=nN_c$. For integral windings Z_s is by definition a multiple of 2p and $N_c= Z_s$ so the first slotting harmonic is involved in the largest cogging torque harmonic as expected. However, this is no longer true for synchronous machines with fractional slot winding.

3.2.3 Case of positive wavenumber (r>0)

The lowest positive wavenumber of magnetic forces is given by [17]

$$r_{\min} = GCD(Z_s, 2p) = M_c \,. \quad (12)$$

The associated force waves are

$$f = nN_c f_R \pm 2f_s \text{ and } r = 0 \pm r_{\min} \,. \quad (13)$$

4. Optimal skew

The optimal stator skew to cancel the pole / slot interaction pulsating harmonics is at no-load

$$\alpha_{ssk} = \frac{2\pi}{nN_c} \,. \quad (14)$$

For the first rank (n=1) and for integral winding, the optimal average value is therefore one stator slot pitch. According to [18] the optimal stator skew value for SPMSM due to is closer to $\alpha_{ssk} = \dfrac{2\pi}{N_c + p}$ but this is because a limited number of slices has been used in their FEA model. When using a stepped-skew instead of a continuous skew the optimal value becomes

$$\alpha_{ssk,step} = \frac{N_s - 1}{N_s} \alpha_{ssk} \,. \quad (15)$$

Where N_s is the number of slices. In [18] this gives for 7 slices of FEM model an optimal skew value of 6/7 stator slot pitch which has been mistakenly taken as 36/42=36/(36+7).
At full load the optimal stator skew is different because all the harmonics identified previously can now either include the fundamental rotor field or the fundamental stator field. The optimal skew then depends on the load angle as demonstrated in [19].
For stepped skew magnets the same optimal skew angle of (15) applies for the cancellation of r=0 wavenumbers at no-load. However the optimal skew for r>0 is different.

5. Numerical validation

5.1 Presentation of MANATEE software

MANATEE® [12] initially stands for Magnetic Acoustic Analysis Tool for Electrical Engineering: it is a commercial simulation software dedicated to the fast electromagnetic design of electrical machines, including the evaluation of 3D electromagnetic forces, vibrations and acoustic noise due to Maxwell forces at variable speed.

It is an integrated multiphysic tool, the simulation process is summarized in Figure 5. Assuming a weak coupling between structural mechanics and electromagnetics the electrical currents are first calculated using equivalent electrical circuits. Based on rotor and stator current waveforms, the electromagnetic module calculates the airgap flux distribution using subdomain models, which are as accurate as finite element method and as fast as analytical models. The structural module consists in projecting the resulting Maxwell stress on the stator or rotor structure, and evaluating the dynamic deflections. The acoustic module finally calculated the radiated sound power and pressure levels.

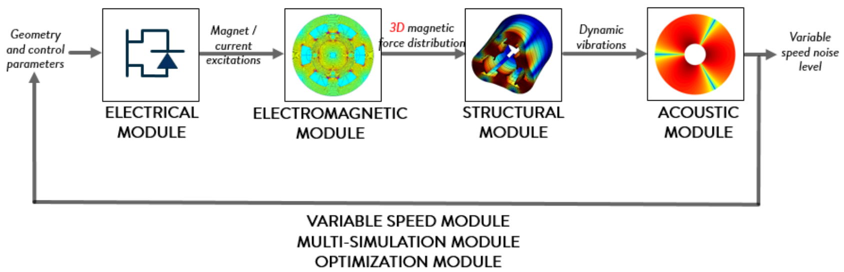

Figure 5: MANATEE software simulation workflow [12].

5.2 Simulation results

In this example a sinusoidally-fed 4-pole, 36-slot surface permanent magnet synchronous machine is simulated up to 15000 rpm, including load effect. The rotor skew is done with 7 slices.

The natural frequencies of the lamination are obtained with MANATEE at 7500 Hz for breathing mode (m=0) and 5900 Hz for the 2p circumferential mode (m=2p=4). The lowest force wavenumber is given by M_c=4. The resulting sound pressure level is shown in Figure 6. The following resonances with stator lamination can be identified using MANATEE post processing tools:

- mode (0,0) at 6235 rpm is excited by $f=2*N_c f_R = 72 f_R = 36 f_s$
- mode (0,0) at 12470 rpm excited by $f=N_c f_R = 36\ f_R = 18\ f_s$
- mode (4,0) at 11010 rpm excited by $f=N_c f_R - 2*fs = 16\ f_s$
- mode (4,0) at 8795 rpm excited by $f=N_c f_R + 2*fs = 20\ f_s$
- mode (4,0) at 5180 rpm excited by $f=2N_c f_R - 2*fs = 34\ f_s$
- mode (4,0) at 4628 rpm excited by $f=2N_c f_R + 2*fs = 38\ f_s$
- mode (4,0) at 3100 rpm excited by $f=3N_c f_R + 2*fs = 56\ f_s$

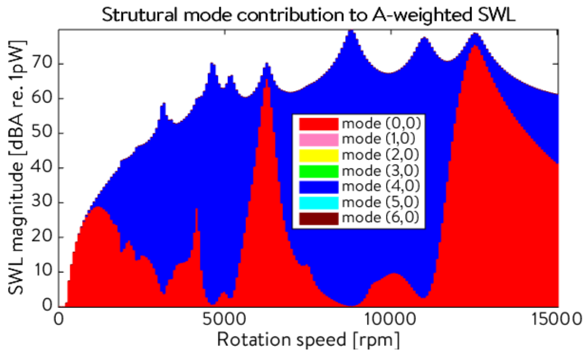

Figure 6: sound power level due to Maxwell excitations at variable speed (simulation time a few seconds on a laptop).

In Figure 7 the effect of stator skew is presented. The optimum value of 1 stator slot pitch is found for both r=0 and r=4.

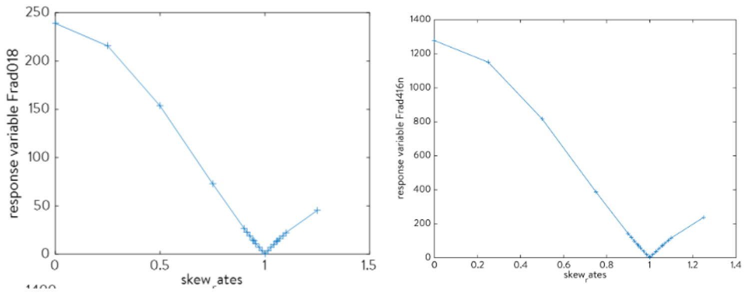

Figure 7: effect of stator skew on the pulsating force wave r=0 f=18f_s (left) and rotating force wave r=4 f=16f_s (right) at no-load.

In Figure 8 the effect of the rotor skew is presented. One can see that the optimal rotor magnet inclination angle is different when trying to minimize cogging torque and acoustic noise. This is because rotor mmf harmonics creating r=0 standing force waves are different from the ones creating r=4 rotating force waves and responsible for the majority of acoustic noise is this case. For cogging torque minimization the numerical optimal skew is exactly 0.85 stator slot pitch in agreement with theory (6/7).

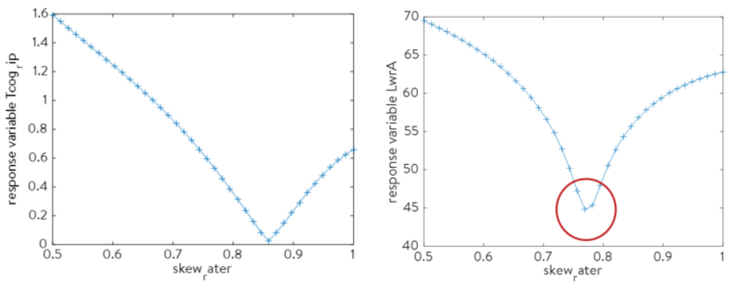

Figure 8: effect of rotor stepped-skew on cogging torque (left) and acoustic noise at 1500 rpm (right).

6. Conclusions

It is shown that the optimum stator or rotor skew value to minimize noise and vibration due to magnetic forces in PMSM is not necessarily one slot pitch contrary to usual design rules. This is because the one slot pitch rule focuses on the reduction of r=0 force wavenumbers at no-load. Moreover, the reduction of noise and vibration levels is not correlated with the minimization of cogging torque or torque ripple. Besides that, the optimum skew to reduce acoustic noise due to magnetic forces depends on the load state.

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