Effect of the load angle on radial and tangential magnetic forces in Permanent Magnet Synchronous Machines

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This paper studies the effect of the load angle on the harmonics of magnetic forces in surface permanent magnet synchronous machines (SPMSM). The purpose is to determine a load angle that both maximize torque and minimize the other harmonic forces to potentially reduce the audible noise and vibration level due to Maxwell forces. The study is carried on the SPMSM with a low torque/high speed machine with 6 stator slots and 2 pole pairs (high speed compressor application). Subdomain semi-analytical modeling technique is used to calculate magnetic forces and favorably compared to finite elements. It is shown that the load angle which maximizes torque does not maximize nor minimize any harmonic magnetic forces.

Index Terms—*Permanent Magnet Synchronous Machines, Electromagnetic forces, Acoustic noise, Vibrations*.

I. INTRODUCTION

The effect of the current load angle (I_d/I_q distribution) on the harmonics of magnetic forces, *a fortiori* on noise and vibrations, is not well known. In some topologies such as concentrated winding machines, harmonics of magnetic forces created by stator armature field may interact with the harmonics created by rotor field, creating low wavenumber harmonic forces responsible for noise and vibration issues. This interaction may decrease or increase the noise level depending on the stator current phases, and therefore on the load angle.

In [1], the harmonic magnitude of the radial force due to stator slotting is maximal for a maximal positive Id and minimal for a minimal negative Id. The machine studied is a synchronous machine of a wind turbine with many pole pairs and a short-pitched distributed winding. However, the effect on the pulsating harmonics (varying in time but not in space) of force as well as the distinction between harmonics of radial and tangential forces are not studied. The tangential force may be responsible for additional noise and vibrations with the yoke deflection due to teeth bending.

In [3], it is analytically shown that the mean radial force is linked to the square of both I_d and I_q currents, while the mean tangential force linearly depends on the I_q current. However, these conclusions only deal with the average value of the forces (null wavenumber).

Besides, current harmonics injection in d and q axis can reduce the pulsating harmonics of radial force [4]. However, this study focuses on current harmonics and not on the load angle of the fundamental current.

In [5], the load angle effect on force harmonics is studied on a wound rotor synchronous machine, and it is shown that the 0^{th} wavenumber (averaged magnetic forces) time harmonics have different variations with the load angle (monotonic or with a maximum). The authors conclude that the load angle can potentially change the magnitude of some force harmonics.

In [6] the effect of I_d and I_q current is studied on two types of magnetic harmonic radial forces of wavenumber 0 and $2p = 10$. It is shown that the flux weakening region (high negative I_d).

To the authors' knowledge, no paper presents the global effect of the load angle on both radial and tangential harmonics of magnetic forces in surface permanent magnet synchronous machines (SPMSM). This paper more particularly aims at comparing the behavior of two different SPMSM in search for general conclusions.

The studied machine, which will be referred as SPMSM *6s4p*, is a surface mounted permanent magnet synchronous machine (SPMSM) with a concentrated winding, $Z_s = 6$ stator teeth and $2p = 4$ poles. This machine is dedicated to high speed and low torque applications (compressor application from [6]).

II. THEORETICAL ANALYSIS OF MAGNETIC FORCES

A. *Principle*

In rotating electrical machines, magnetic forces that are responsible for noise and vibrations are mainly due to the interaction of electromagnetic fields in the air gap. They are usually called Maxwell forces. As the electromagnetic field, they vary over space and time. Hence, harmonics of magnetic forces are defined by their direction (radial or tangential component), frequency f, wavenumber r and rotating direction. These parameters are obtained using Fourier series expansion as follows:

$$\underline{\boldsymbol{A}}(t,\alpha) = \sum_{f,r} \boldsymbol{A}_{fr} \exp(i(2\pi f t - r\alpha + \varphi_{fr}))$$

The steady component of the forces, at frequency $f = 0$ and wavenumber $r = 0$, represents the mean electromagnetic torque (or mean tangential force) and the mean radial force which has no effect on noise and vibration harmonics but only on structural mechanic as static loads. Harmonics with $r = 0$ are called pulsating harmonics. The pulsating harmonics of the radial force generate a radial deflection of the stator yoke and may produce noise. The pulsating harmonics of the tangential force cannot directly generate noise due to the tangential vibrations of the stator circular yoke. However, for certain yoke's boundary conditions, the tangential forces r=0 may induce radial deflection of the yoke and so generate noise. Harmonics of wavenumber $r = 1$ induce an unbalanced magnetic pull (UMP) which can excite a rotor bending mode. Finally, harmonics of radial or tangential forces whose wavenumber is greater than $r = 2$ can both generate radial yoke deflection and potentially induce noise.

B. Theoretical application

Under open-circuit condition, harmonics of radial and tangential forces have the same frequency and wavenumber [9]. For SPMSM *6s4p*, the following harmonics with non-zero frequency are:

- $r = 2p = 4$; $f = 2f_s$: « fundamental » rotating force wave linked to the square of the fundamental of the magnetic field,
- $r = Z_s - 2p = 2$; $f = 2f_s$: harmonic of force due to the presence of stator slotting harmonics in the magnetic field. The smallest wavenumber is given by $r = GCD(Z_s, 2p) = 2$,
- $r = 0, f = 6f_s$: pulsating harmonic of force whose frequency is given by $f = LCM(Z_s, 2p)f_s/p$. The tangential component is the cogging torque dominant frequency.

Under load situation, the spectrum of magnetic forces presents the same frequencies and wavenumbers, only their magnitude is modulated. In such case, there is either a constructive or destructive interference between the harmonics of forces due to the stator armature field and the ones due to the rotor magnet field. The interference depends on the current load angle, which determines the phase shift between the fundamental component of both stator and rotor magnetic fields.

III. COMPUTATION OF MAGNETIC FORCES

A. MANATEE software

The magnetic forces are numerically computed using MANATEE software [8] [9] to validate the previous theoretical study and then show the impact of the current load angle on the magnetic forces and induced noise and vibrations level.

The operating point is first obtained using an Electrical Equivalent Circuit. The flux density is then computed using the the semi-analytical subdomain method (SDM) [10]. This method gives as accurate and much faster results than the finite element method [11] under infinite permeability of material assumption. The flux density is directly obtained under the form of Fourier series, for each time-step independently. Then the radial and tangential magnetic forces are computed using the Maxwell stress tensor along a circular path at the middle of the air gap.

The vibroacoustic model of MANATEE is based on semi-analytical models. MANATEE results are validated thanks to finite elements analysis and experimental measurements on a SPMSM with concentrated windings [12].

B. Computation of the air gap flux density

In a first step, the open-circuit magnetic field is computed (NL: No-Load), then only with stator armature field (AL: Armature-Load) and finally with full load (FL) with current load angle $\varphi = 90°$. Figure present the radial and tangential air gap flux densities for each operating case at the initial rotor position. The mean squared error between SDM in (blue) and finite elements (in red, using the automated coupling between MANATEE and FEMM software) is less than 1%.

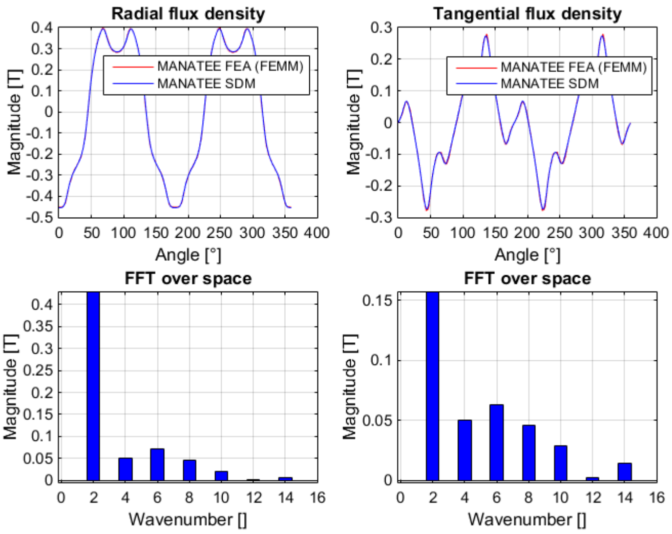

Figure 1: Radial and tangential air gap flux densities at NL (SPMSM *6s4p*) over space a the initial rotor position.

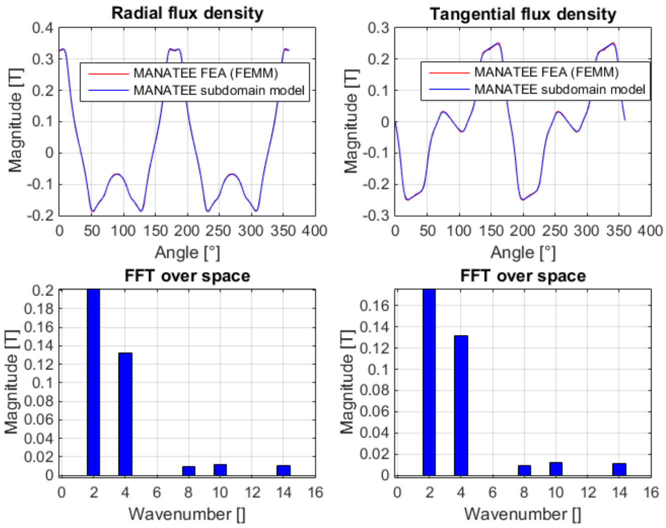

Figure 2: Radial and tangential air gap flux densities at AL (SPMSM *6s4p*)

The magnetic flux density is computed in the air gap for 1024 angular steps and 2048 time steps for the SPMSM *6s4p* and for 41 different values of load angle. This fine time and space discretization enables to accurately compute the 2D Fourier Transform (FFT2D) of the magnetic forces, and deduce the frequency and the wavenumber of each harmonic of forces. The subdomain method computation takes around 58 seconds for SPMSM *6s4p* when sweeping all the load angles.

C. FFT2D computation of the magnetic forces

As said previously, the space and time distribution of the magnetic forces is computed using the Maxwell stress tensor applied to the air gap flux density. The following study is first made with maximum I_d current (current angle $\varphi = 0°$).

The Figure 3 and 4 show the magnitude of the FFT2D radial and tangential magnetic forces. Red and black crosses represent the natural frequencies of the resonance modes of the stator that can be potentially excited by the harmonics of forces on the same line (meaning with the same wavenumber).

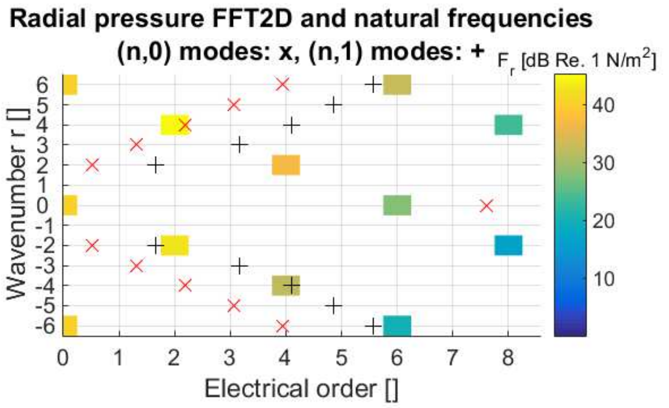

Figure 3: FFT2D magnitude of the air gap **radial magnetic pressure** (Id maximum, Iq=0)

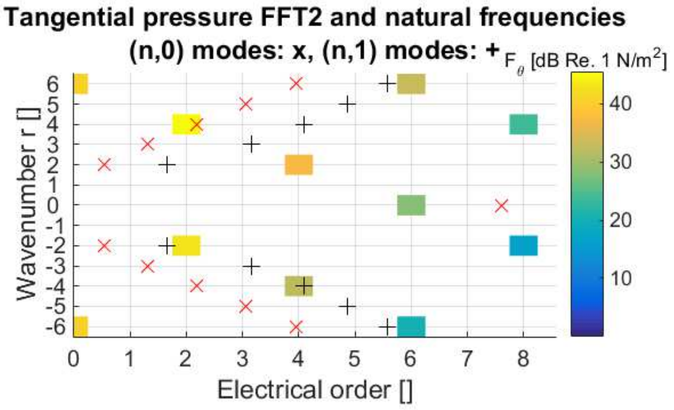

Figure 4: FFT2D magnitude and phase angle of the air gap **tangential magnetic pressure** (Id maximum, Iq=0)

As frequencies are expressed in terms of electrical orders, they must be multiplied by the electrical fundamental frequency f_s to obtain the value in Hz. Frequencies of electrical order greater than 8 are not represented on the graph for more readability.

Besides, there are two harmonics of wavenumber $r = \pm 6, f = 0$ with same magnitude at the top and bottom left hand corner of the Figure 3 that were not predicted in the theoretical study. However, these harmonics are in opposition of phase given by the FFT2D. It means that both harmonics cancel each other.

As the current angle is $\varphi = 0°$, the average torque is null, meaning that the mean tangential force is null as it can be seen on the Figure 4:. This particular case also shows that frequencies and wavenumbers are the same for both radial and tangential forces.

When the rotation speed changes, force harmonics of wavenumber $r = 2$ or 4 may excite stator resonances (meaning match a red cross). The radial deflection of the stator yoke may be caused by harmonics of either radial (due to tension/compression of teeth) or tangential forces (due to teeth bending).

IV. EFFECT OF LOAD ANGLE

A sensitivity study on the current load angle is carried out for both SPMSM. The current load angle varies between 0 to 360°, while the total current $I_0 = \sqrt{{I_d}^2 + {I_q}^2}$ magnitude remains constant. The mean radial and tangential forces (similar to the average torque) are also computed, even if they have no effect on the noise and vibrations level of the machine because their frequency is null.

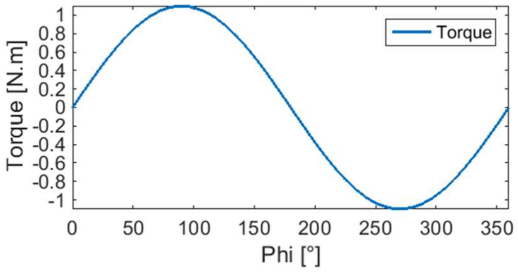

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Figure 5: Torque in function of current load angle for SPMSM *6s4p*

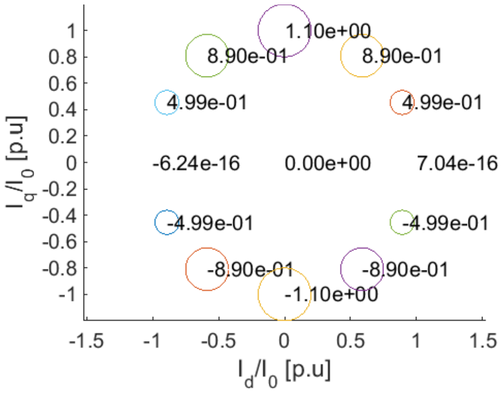

Figure 6: Polar representation of the torque (r=0, f=0, **tangential**) in function of Id/Iq for SPMSM *6s4p*

Harmonics of magnetic forces under open-circuit (NL) condition are preliminary computed to show how stator and rotor contributions interfere depending on the value of the current load angle at full load (FL).

A. Sensitivity study :

The torque is shown in Figure 5. It is maximum for a load angle of 90° as expected for SPMSM. The waveform is a pure sinus as there is no reluctant torque in SPMSM. The polar representation used in the afterwards is the same as the one used in [2]. The polar plan is the I_d/I_q plan, and both I_d and I_q have been normalized with the current magnitude I_0.

The harmonics variation of the radial force and the polar representation for one harmonic ($f = 2f_s, r = 2$) are presented in Figure 1 and Figure 8. Both « FL » and « NL» variations are represented in the same color for the same harmonic. « FL » is in plain line whereas « NL » is in dashed lines.

In vibroacoustic studies, noise and vibrations are related to the magnetic forces according to $20log10(F)$ in dB, where F is the magnitude of the harmonic in N/m^2. Hence a multiplication by a factor 2 for the magnitude of the harmonic results in an increase of 6 dB.

The following conclusions can be made for SPMSM 6s4p:

- The mean radial force is maximized for maximal I_q ($\varphi = 180°$) and does not cancel for null I_q ($\varphi = 0°$);
- The « fundamental » of the radial force ($f = 2f_s, r = 4$) is maximal for $\varphi = 180°$ and minimal for $\varphi = 45°$;
- The forces linked to the fundamental magnetic field ($f = 0, r = 0$) and ($f = 2f_s, r = 4$) reach their minimum for a current angle near 45° and 55° ;
- The pulsating radial force at $6f_s$ is in phase opposition compared with the other harmonics and is minimized for $\varphi = 180°$;
- The current load angle which maximizes torque does not minimize nor maximize any harmonic of radial forces;
- The current load angle induces up to 6 dB variation on the slotting harmonic;
- The effect of current load angle is the most significant on the pulsating radial force (20 dB).

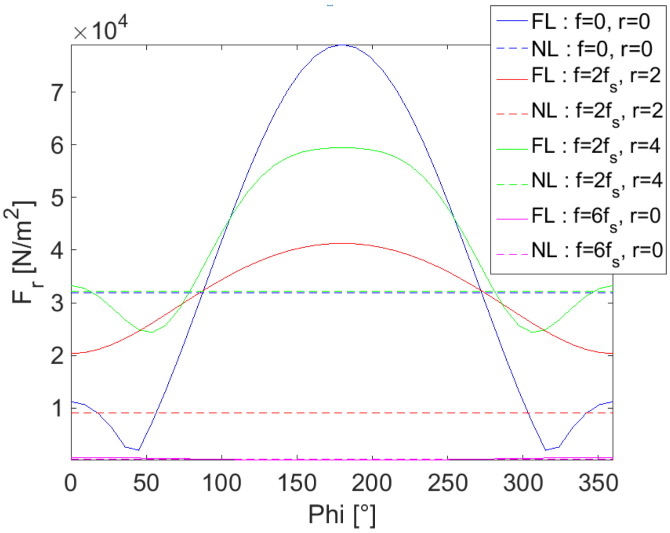

Figure 1: Variation of harmonics magnitude of **radial pressure** in function of the current load angle for SPMSM *6s4p*

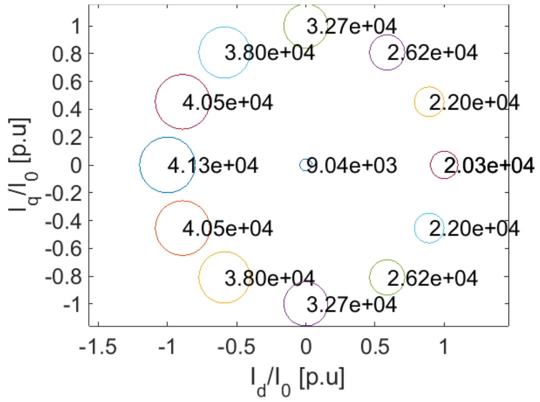

Figure 8: Polar representation of harmonics of **radial pressure** in function of Id/Iq for SPMSM *6s4p*

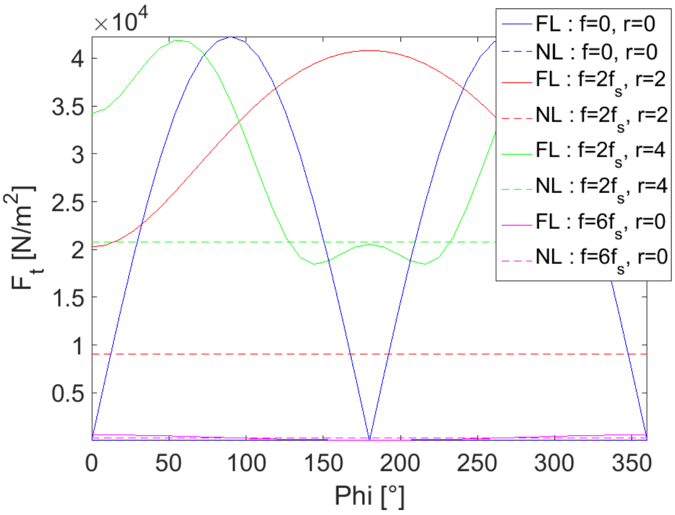

Figure 9: Variation of harmonics magnitude of **tangential pressure** in function of the current load angle for SPMSM *6s4p*

The results about the harmonics of the tangential force are presented in Figure 9. It can be seen that:

- The « fundamental » of the tangential force ($f = 2f_s, r = 4$) is maximal for $\varphi = 60°$ and minimal for $\varphi = 150°$, so not correlated with the fundamental of the radial force;
- The pulsating harmonic of tangential force at $6f_s$ is minimized for $\varphi = 180°$, as well as the pulsating harmonic of radial force;
- The slotting harmonic ($f = 2f_s, r = 2$) and the pulsating harmonic ($f = 6f_s, r = 0$) of tangential force are in phase with the corresponding harmonics of radial force, and they are maximal for $\varphi = 180$;
- The current load angle which maximizes torque does not minimize nor maximize any harmonic of tangential force, as for the radial force;
- The effect of current load angle is the most significant on the pulsating tangential force, as for the radial one and so has a large influence on torque ripple;

V. CONCLUSION

The effect of current load angle on radial and tangential magnetic forces including mean value, fundamental and harmonics is studied an small highspeed SPMSM using MANATEE, a fast simulation environment dedicated to the electromagnetic and vibroacoustic design of electrical machines. The results show that it is quite difficult to find a general rule to choose the current load angle such as to maximize torque and minimize the harmonics of magnetic forces.

However, in this studied cases, it can be seen that the current load angle which maximizes torque does not minimize nor maximize any harmonic of magnetic forces. For this SPMSM *6s4p*, the pulsating and slotting harmonics are the most influenced by the variation of the load angle. Besides, the load angle which minimizes the pulsating harmonic differs from the one which minimizes the slotting harmonic.

VI. REFERENCES

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