# Spread spectrum strategies study for induction motor vibratory and acoustic behavior

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***Abstract* –This paper investigates the vibratory and acoustic behavior of the induction motor when fed by a power converter under PWM. The mechanical, electromagnetic and vibratory models of the stator vibrations generation process are first described. Then, different spread spectrum strategies are experimented and compared with natural sampling PWM as a reference strategy. The harmonics caused by the PWM can become of prime importance when matching a mechanical resonance: thus, the power converter design has a great influence on the motor vibratory and acoustic behaviour.**

## NOMENCLATURE

p : number of pole-pairs (p=2),
fs : frequency of the supply,
Zs : stator slots number,
Zr : rotor slots number,
Rc : average radius of the stator,
Ra : stator inner radius,
hcul : height of the stator yoke,
m : mode number,
f : radial force harmonics frequency,
ρ: density of stator material,
E : modulus of elasticity ( N/m2 ),
ξa : damping coefficient.

## I. INTRODUCTION

The vibration analysis of electrical machines is a rather old problem which was motivated by the stator high acoustic noise emission. During the 40s and 50s, it was deeply studied by various researchers with an interesting synthesis made by JORDAN [1]. LIWSCHITZ published an excellent paper [2] with an extended list of references on the subject in 1942. ALGER wrote one of the main publications on the subject in 1951 in the first edition of [3]. However the problem is still a topical issue [4], [5], [6]. The vibrations of electro-mechanical systems are due to exciting forces, some of which are of magnetic origin. Other sources such as to aerodynamics or bearings are not considered in this study.

Acoustic noise generated by the association of an electric machine and its converter has now become of prime importance for choosing an electric drive. This problem has been studied since the wide use of the electronic converters. One of the first references on the subject is due to P.L. TIMAR in 1977 [7] and some books [8] have included a synthesis on the problem.

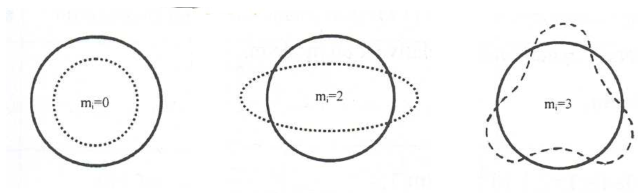

Fig. 1. Vibration mode example.

A more recent paper dealing with acoustic noise of induction motor and the influence of power converter is due to Lo [9]. The most efficient method for reducing induction motor noise is the use of high switching frequencies (above 16 kHz). However it imposes high stress of the switches, and such switching patterns can be inapplicable like in high power applications. In that case, spread spectrum strategies can be used for acoustic noise reduction. The most known of them uses randomization of pulse placement or switching frequency [10], [11], [12] and [13].

This paper explains first the generation process of stator vibrations with help of electromagnetic, mechanical and vibratory models. Then, the experimental setup and the PWM strategies under investigation are presented. The natural sampling PWM strategy (sinus-triangle comparison) is taken as the reference strategy. Two spread spectrum strategies belonging to the FMPWM family where the carrier frequency is modulated are studied: the first one is randomly modulated (RMPWM), whereas the second one is sinusoidaly modulated (SMPWM). Finally, experimental results are shown and discussed.

## II. INDUCTION MOTOR VIBRATION

The stator vibration is the result of some exciting radial forces, and becomes of prime importance when it is amplified by a mechanical resonance.

### *A. Mechanical Model*

According to its spatial mode order (Fig. 1), a deformation arises in various forms [1], [8]. For example, the force wave responsible for the oval deformation corresponds to mode number 2.

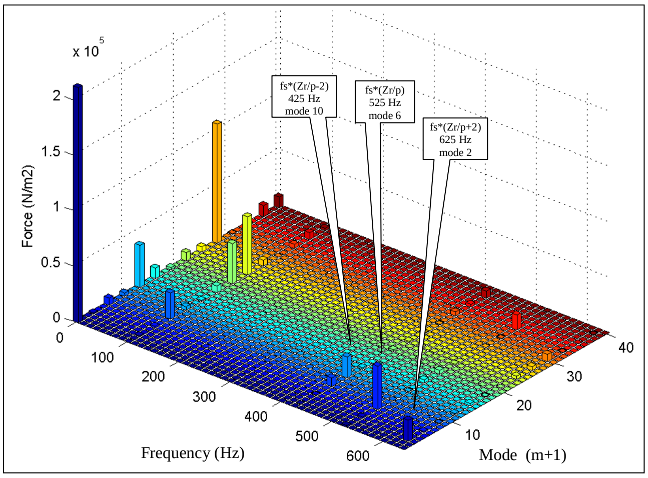


Fig. 2. Harmonics of surface force in space and time.

Our studies, as the previous ones, showed that the most significant modes 'm' lie between 1 and 4 (cf rq sur mode 0). The mode number 2, as seen later, can be especially dangerous when excited.

Each mode shape is associated with a mechanical resonance frequency. The rather simple formulas given by [14] make it possible to easily find the resonance frequencies of each stator mode (yoke + teeth). For m=0 mode, the resonance frequency of the stator is given by:

$$F_0 = \frac{1}{2\pi}\sqrt{\frac{E}{\rho R_c^2 \Delta}}, \qquad (1)$$

Where:

$$\Delta = \frac{weight.of.yoke + total.weight.of.teeth}{weight.of.yoke}$$

For the m>1 modes, the stator resonance frequencies for each mode is given by:

$$F_m = \frac{H_{cul}}{2\sqrt{3}} \frac{m(m^2-1)}{R_c\sqrt{m^2+1}}, \qquad (2)$$

The analytical mechanical model was validated numerically by ANSYS (F.E.M) calculations and experimentally. A part of these results are presented in Table 1.

*B. Electromagnetic model*

The method for calculating the flux density (b) in the air-gap is based on the product between the permeance of air-gap (P) and the magnetomotive force (mmf):

$$b = P.\ mmf \qquad (3)$$

The goal is to determine the analytical expressions of the magnetomotive force and the permeance, by considering the different harmonics due to the winding and the slot configuration, responsible for electromagnetic noise in electrical machines [15]. The permeance can be determined using simple assumptions:

- the magnetic circuit has a high permeability and a linear characteristic,
- the tangential component of the air-gap flux density is negligible compared to the radial one.

The radial force per unit area is proportional to $b^2$. Its two-dimensional Fast Fourier Tranform (in space modes and frequencies) is computed thanks to the analytical model. It is presented in Fig. 2 for the tested motor with sinusoidal current.

TABLE I
FREQUENCY AND MODE NUMBER RESONANCE

| Mode number | Analytical Method (Hz) | Finite Element (Hz) | Shock Method (Hz) | Sinus Method (Hz) |
|---|---|---|---|---|
| 0 | 14859 | 14656 | O. R. | O. R. |
| 1 | 0 | 0 | 1200 | 1273 |
| 2 | 2478 | 2364 | 2400 | 2423 |
| 3 | 6396 | 6473 | 6100 | 6210 |
| 4 | 12028 | 11898 | 11700 | O. R. |

*C. Vibratory Model*

In this study, vibrations are the consequences of excitation of the mechanical system by electromagnetic forces. Once the forces applied to the stator have been determined, we can study the vibrations which correspond to these deformations. Mode 1 has no radial deformation, only a longitudinal one. It depends principally of the mechanical design quality, thus it is not taken into account in our study.

For the mode m=0, the amplitude of the static deformation of the stator is given by:

$$Y_{0\,stat} = \frac{R_a R_c F_{rmw}}{EH_{cul}}, \qquad (4)$$

Y0stat : Amplitude of the static deformation force,
Frmw : Amplitude of the static force ( N/m2 ).

For the modes, higher than 1, the stator yoke is subjected to an inflection effort given by the following equation:

$$Y_{mstat} = 12\frac{R_a R_c^3}{EH_{cul}^3}\frac{F_{rmw}}{(m^2-1)^2}, \qquad (5)$$

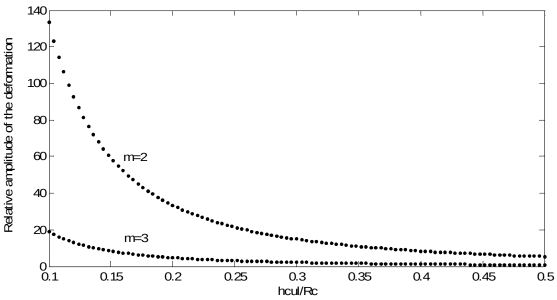

Fig. 3. Relative amplitude of deformation.

According to the relations (4) and (5), the static amplitudes for similar geometrical types increase with geometrical parameters size. Moreover, the amplitude of the deformation Y0stat is slightly influenced by the reinforcement of the stator (relative thickness), considering thatY0stat decreases only linearly with Rc/hcul. To estimate the importance of a vibration form, it is relevant to compare the m-th order static amplitude with the zero order one for the same radial force (6). For the modes m > 1, this ratio is given by:

$$D_{mstat} = \left| \frac{Y_{mstat}}{Y_{0stat}} \right| = \frac{12}{(m^2-1)^2} \left( \frac{R_c}{H_{cul}} \right)^2 , \qquad (6)$$

Dmstat depends on the order of the m-th force mode and the rigidity of the stator which is characterized by the relative height of stator yoke (hcul/Rc) [1]. For example, the values of Dmstat versus the mode order are showed in Table 2 for hcul/Rc=0.2.

TABLE II
AMPLITUDE RATIO OF DEFORMATION VERSUS MODE NUMBER

| m | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $D_{MSTAT}$ | 36.2 | 5 | 1.4 | 0.5 |

According to Table 2, the elliptic deformation (m=2) is more dangerous than the highest order deformations, especially when hcul/Rc is smaller (Fig. 3). After having computed the resonance frequencies the structure Fm, the dynamic deformations are calculated. They express for modes m>1 as:

$$Y_{md} = \frac{12\, R_a R_c^3 F_{rmw}}{E H_{cul}^3 (m^2-1)^2 \sqrt{(1-(\frac{f}{F_m})^2)^2 + (2\xi_a \frac{f}{F_m})^2}} , \quad (7)$$

where ξa is the damping coefficient. It can not be expressed theoretically, however [1] considers that for an induction machine this one is between 0,01 and 0,04.

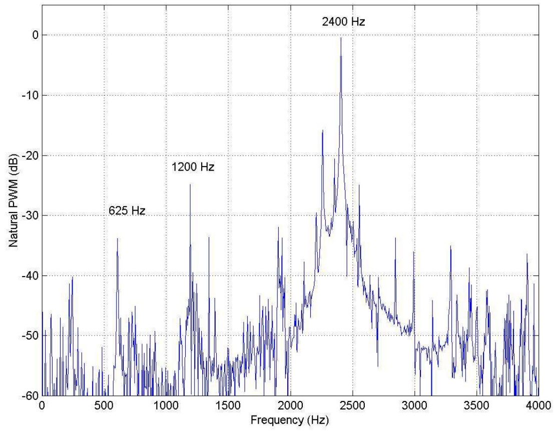

Fig. 4. Vibration for natural PWM.

The highest radial stator vibrations occur when the excitation force harmonics (f) are equal or close to one of the above frequencies Fm; this is only possible when there is a coincidence of the spatial modes.

In Fig. 4 which represents blabla, the peak at 625 Hz (fs[Zrp +2]) corresponds to the excitation mode 2 which is not dangerous in this case because it is far from the mode 2 resonance frequency (2400 Hz) of the induction motor.

## III. PWM STRATEGIES AND IMPLEMENTATION

The use of PWM to feed the induction machine implies increasing the exciting force harmonics content. The harmonics coming from the fundamental of current and the machine design remain, like the peak at 625 Hz. But a lot of harmonics coming from PWM are susceptible to excite the mechanical resonances around 1200 Hz and 2400 Hz for instance. Figure 4 shows a vibration spectrum for natural PWM with 1800 Hz switching frequency. We can see 625 Hz and 1200 Hz peaks but the most important peak is the 2400 Hz one because an important exciting force of mode number 2 is generated at 2400 Hz by the PWM [10].

The goal of this study is to compare at first spread spectrum strategies with natural PWM then RMPWM and SMPWM together. Figure 5 shows voltage spectra with only 4 peaks for natural PWM but with relatively high level. The two spread spectrum strategies have more harmonics with lower level.

In order to compare experimentally these different methods, they have been implemented on a TMS320F240 Spectrum Digital DSP card with help of C Language as shown Fig. 6. The studied machine is an induction motor mechanically connected to a load machine, and a Bruël & Kjaer accelerometer sensor gives vibration measures through a Yokogawa 12 bits data acquisition system.

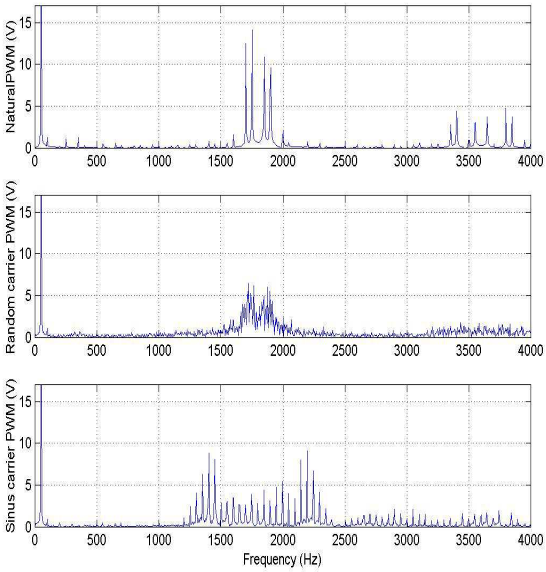


Fig. 5. Voltage spectra.

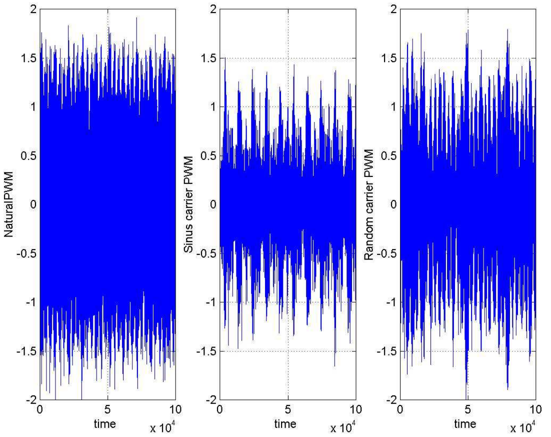


Fig. 7. Vibration signal.

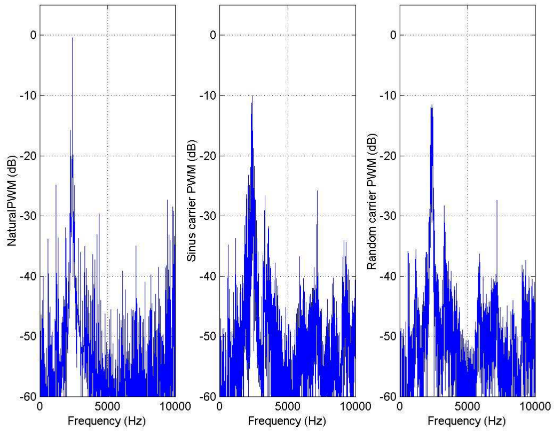


Fig. 8. Vibration spectra.

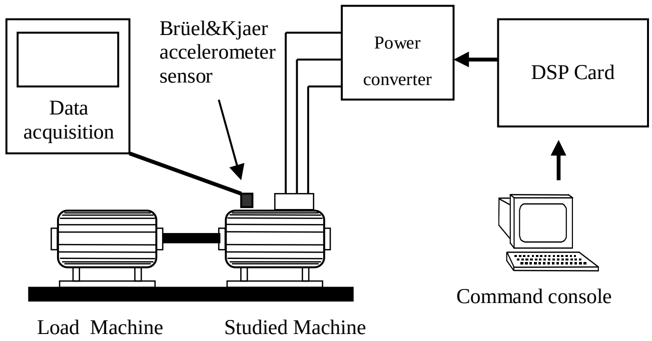


Fig. 6. Experimental implementation.

## IV. EXPERIMENTAL RESULTS AND DISCUSSION

### *A. Vibratory results*

Figure 7 and 8 show respectively the temporal signal of stator vibration and its spectrum for natural PWM, SMPWM and RMPWM. Spectra are presented for a large frequency range; the harmonics after 10 kHz are very weak. Spread spectrum strategies have the same modulation range of switching frequency. We can see in Fig. 7 that the lower amplitude is for SMPWM and the higher for natural PWM but spectral analysis shows other results. In fact, spread spectrum strategies have around 10 dB less than natural PWM for the most important peak with a slight advantage for RMPWM. The shape of RMPWM and SMPWM spectra is almost similar but the sound produced is significantly different. Thus, a deeper study is engaged with several modulation ranges of switching frequency for the two spread spectrum strategies.

Several modulation ranges around the mean switching frequency are tested between ±200 Hz and ±1000 Hz. Figure 9 shows the comparison between RMPWM and SMPWM vibrations with the first modulation range (±200 Hz). Spectra are focused on the most important peak around 2400 Hz. With this first modulation range SMPWM gives bad result with a 7 dB higher level as compared to RMPWM. However, for the ±400 Hz modulation range (see Figure 10), spectra results are very close and the amplitude of SMPWM temporal signal is slightly lower than RMPWM. The other tested modulation ranges almost bring no improvement. So, in our case, the best configuration is the ±400 Hz modulation range.

However, even if some vibrations occur around the same frequency with the same level, the noise produced is different. Figure 11 presents a zoomed view of vibration spectra around the principal peaks for natural PWM, SMPWM and RMPWM with the ±400 Hz modulation range. Natural PWM produces a very big vibration peak at 2400 Hz which corresponds to a monotonic sound. Let us notice that from this point, our work reaches the psychoacoustic field and that some noise characteristics may be better described by words than by numbers.

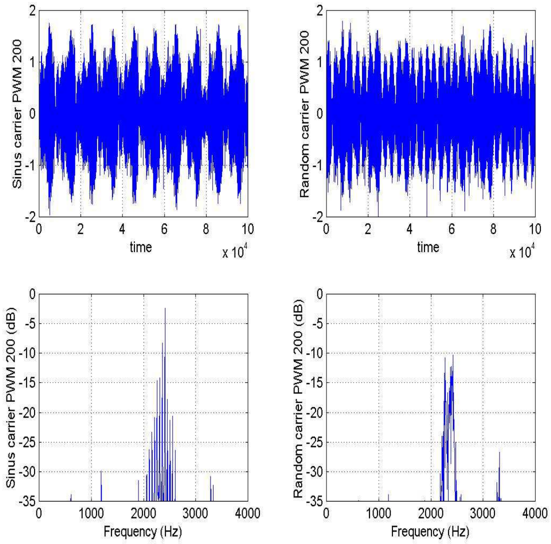


Fig. 9. Vibration for ±200 Hz modulation range.

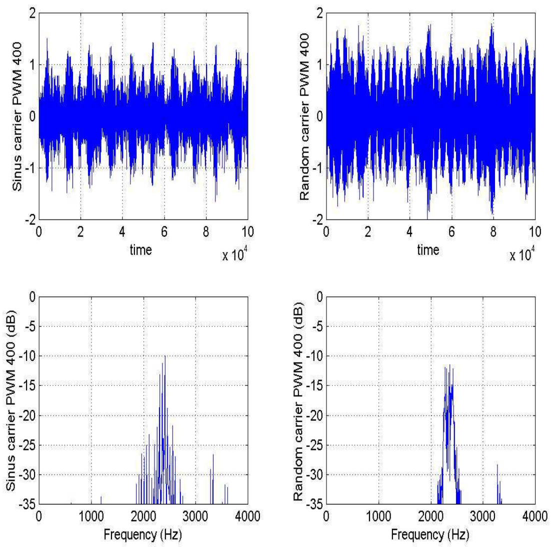


Fig. 10. Vibration for ±400 Hz modulation range.

The monotonic sound is known to be the most unpleasant sound, which is largely confirmed by experiments. SMPWM produces several harmonics lower than for natural PWM. Only three of them are of prime importance: 2310, 2360 and 2410 Hz. They are equally spaced by 50 Hz, the corresponding sound is more pleasant and has lower level than the natural PWM one. The third case (RMPWM) doesn't show dominant harmonics but a frequency area between 2230 and 2440 Hz. The level of noise is quite similar than the last one but it produces a special sound as if there were "sand" in the machine. This sound is not so pleasant to ear and gives an impression of bad quality of the motor. This last point could be very embarrassing for lots of industrial applications.

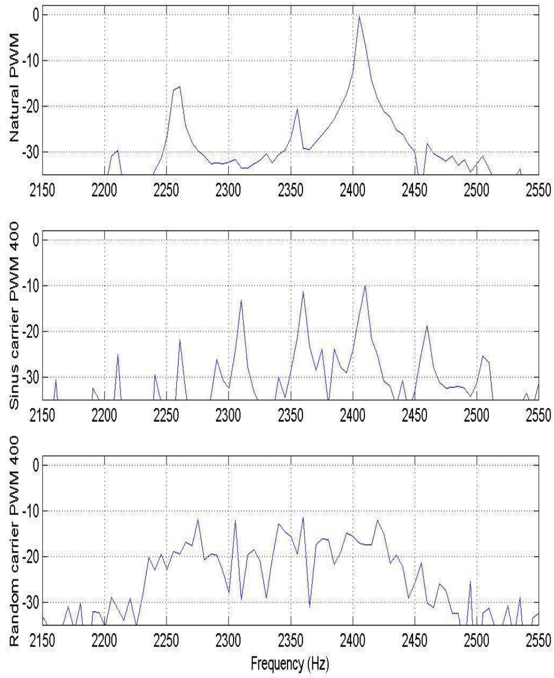


Fig. 11. Vibration zoom.

Therefore SMPWM should be preferred in these cases. The major difference between SMPWM and RMPWM comes from the voltage harmonics repartition (Fig. 5). For SMPWM, the harmonics are spred in all the modulation frequency range (1800 Hz ± 400 Hz). It is not the case with RMPWM, where harmonics are spred with a Gaussian law which implies a smaller efficient modulation frequency range: harmonics are concentrated in the 1700-1900 Hz area and are not equally spaced, which generates an unpleasant sound. However, for the use of SMPWM, great attention must be paid to the modulation range in order to be different enough from natural PWM behaviour (avoiding the greatest harmonics around the extremity of the sideband). The use of an uniform repartition of the harmonics with a limited level should be an improvement.

## V. CONCLUSION

This paper has presented the generation process of stator vibration with help of mechanical and vibratory model versus electromagnetic exciting force. Spread spectrum strategies and natural PWM have been implemented and studied. Experimental results have been shown and discussed for vibration and acoustic point of view.

Spread spectrum strategies have good results as compared to natural PWM when the most important mechanical resonance is excited. So these strategies are very interesting when the mechanical model of the motor is not perfectly known. However spread spectrum strategies produce complex sound which could be unpleasant to ear especially for RMPWM. SMPWM seems to be a good compromise but improvements should be brought by other harmonics repartition to define in the future.

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