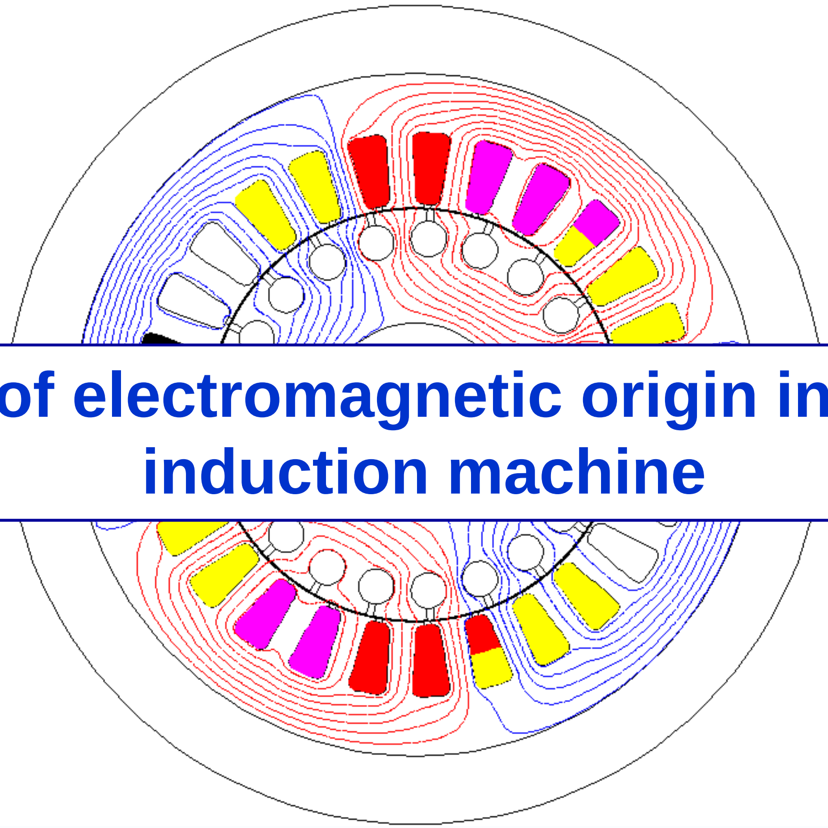
SSD'07 - 4th International Conference on Power Electrical Systems



Acoustic noise of electromagnetic origin in a fractional-slot induction machine

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ADEME

(1) **(2)**

Introduction

How does audible magnetic noise occur ?

- **air-gap electromagnetic energy per unit volume = electromagnetic pressure**

- **enriched spectrum with PWM time harmonics + rotor bars and stator winding space harmonics**

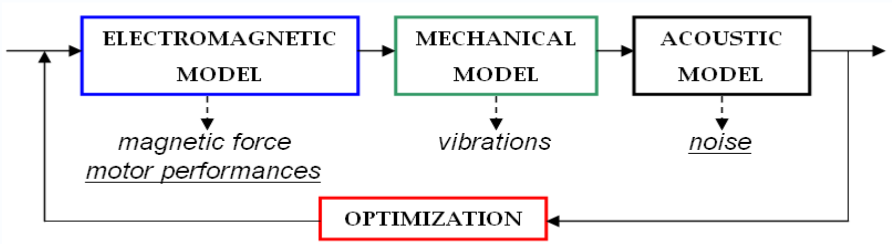
- **radial excitation of stator structure, creating vibrations especially when matching natural frequencies**

Objectives

- **fast prediction of variable-speed electromagnetic noise level of a PWM-fed induction machine (+ optimization)**

- **analytical prediction of main noise spectrum lines**

-> I. Analytical model



-> II. Simulation of a 3-phase fractional-slot induction machine

-> III. Derivation of main noise spectrum lines

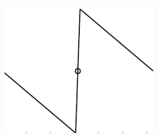
I Analytical model: electromagnetic part

- **Rotor and stator currents are computed using an extended equivalent circuit with both time and space harmonics**

- **Magnetomotive forces are computed using winding function approach with linear rise in slots:**

stator phase currents

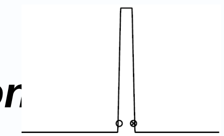
stator turns function



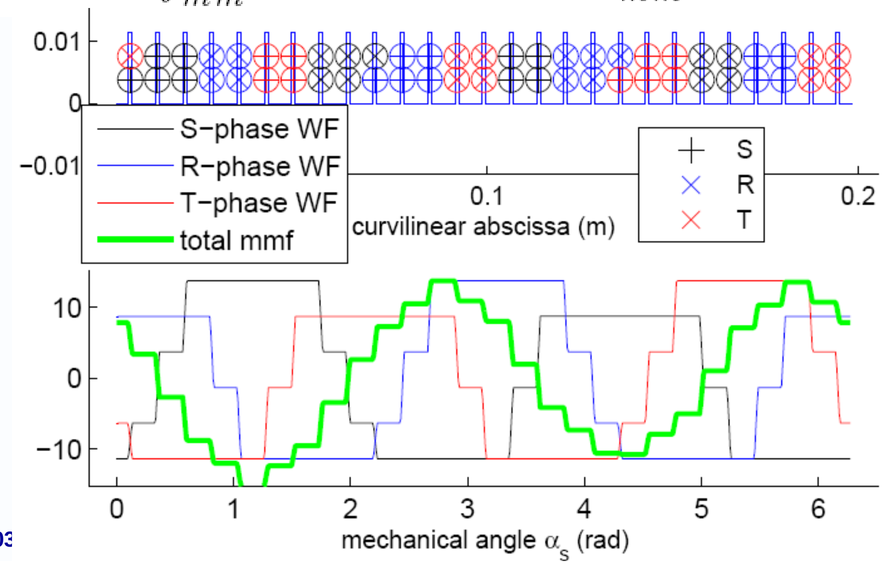
rotor bar currents

$$f_{mm}(t,\alpha_s) = \underbrace{\sum_{q=1}^{q_s} i_q^s(t) N_q^s(\alpha_s)}_{f_{mm}^s} + \underbrace{\sum_{b=1}^{Z_r} i_b^r(t) N_b^r(t,\alpha_s)}_{f_{mm}^r}$$

rotor turns function



Stator mechanical angle

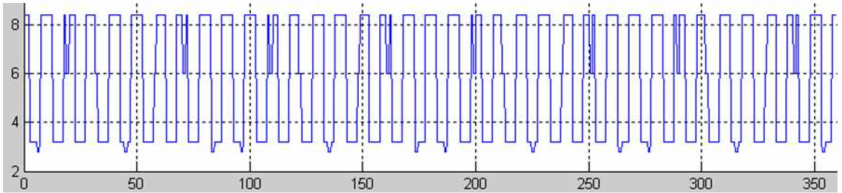


I Analytical model: electromagnetic part

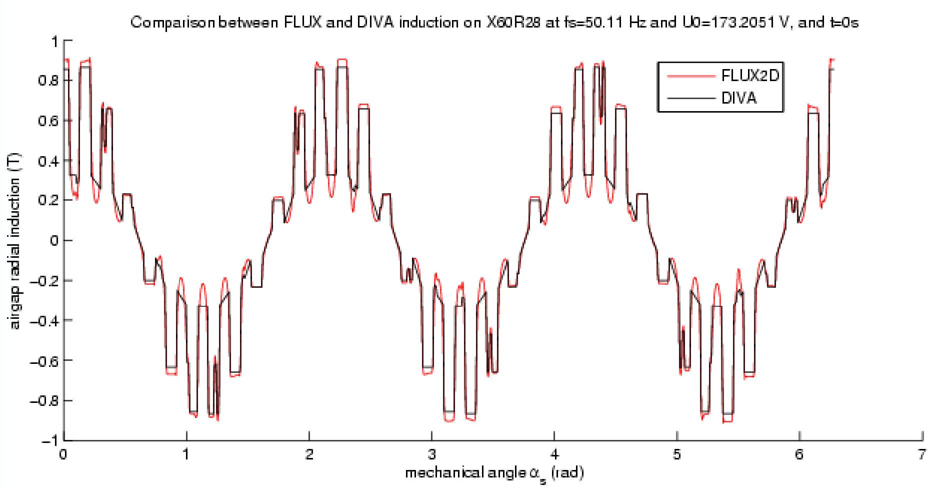
- **Air-gap radial flux density is then computed as:**

$$B_g(t, \alpha_s) = \Lambda(t, \alpha_s) f_{mm}(t, \alpha_s)$$

permeance per unit area



- **Validation by FEM / tests**

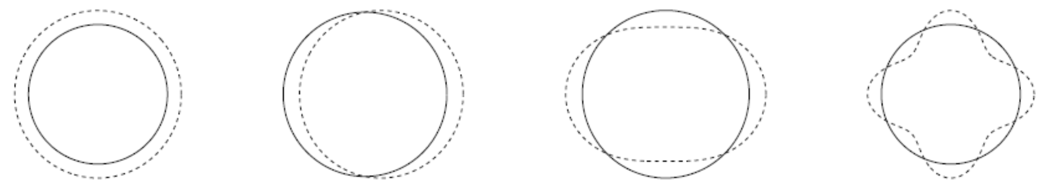


I Analytical model: mechanical part

- Stator vibration is supposed to be generated by Maxwell tensor radial component:

$$P_M = \frac{B_g^2}{2\mu_0}$$

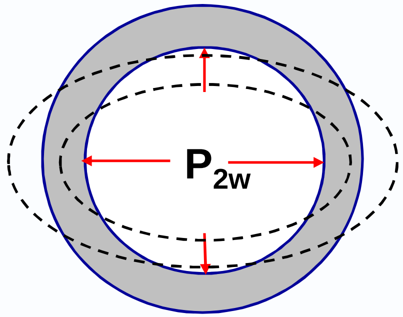
- Pressure is 2D-Fourier developped in pressure lines P_{mw} of spatial modes *m* at pulsation *w.*



m=0 ***m=1*** ***m=2*** ***m=4***

- Static displacements Y^s are computed For instance, for *m* >1

$$Y^s_{m\omega} = P_{m\omega} \frac{12 R_a R_c^3}{E_c h_c^3 (m^2-1)^2}$$



- Dynamic displacements Y^d are computed using a 2nd order filter resonance model:

$$Y^d_{m\omega} = Y^s_{m\omega}[(1 - f^2/f_m^2)^2 + 4\xi_m^2 f^2/f_m^2]^{-1/2}$$

m-th mode natural frequency

m-th mode modal damping

I Analytical model: mechanical part

- **Natural frequencies are analytically computed using a 2D ring stator model**

- **Natural frequencies were validated with FEM/tests**

m	Analytical	2-D FEM	Shock Method	Sinus Method
0	14859	14656	OR	OR
1	1100	ND	1200	1273
2	2478	2364	2400	2423
3	6396	6473	6100	6210
4	12028	11898	11700	OR

I Analytical model: acoustic part

- **Sound power radiated by m-th wave at stator surface is given by**

$$W_m(f) = \rho_0 c_0 S_c \sigma_m(f) < \overline{v_{m\omega}^2} >$$

Modal radiation factor (sphere or infinite cylinder approximation)

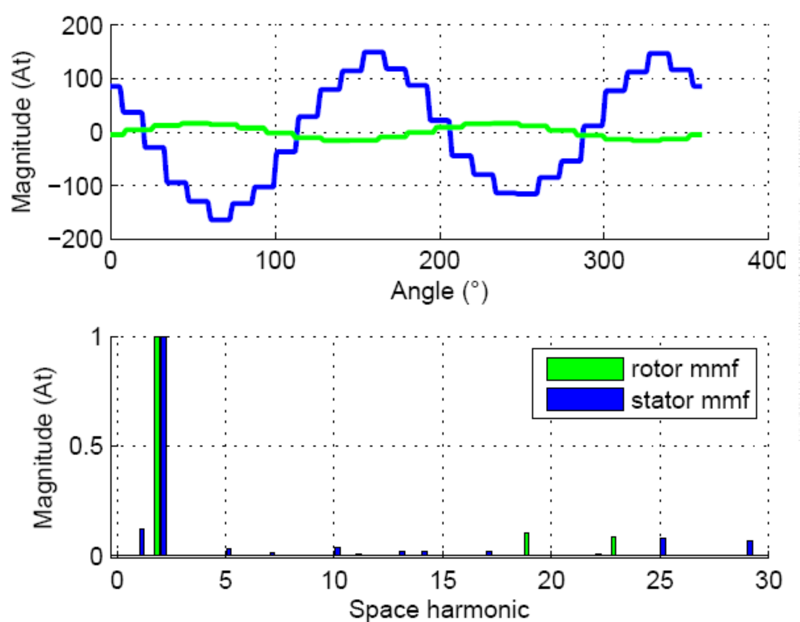
Air velocity proportionnal to Y^d

- **Sound power level is then**

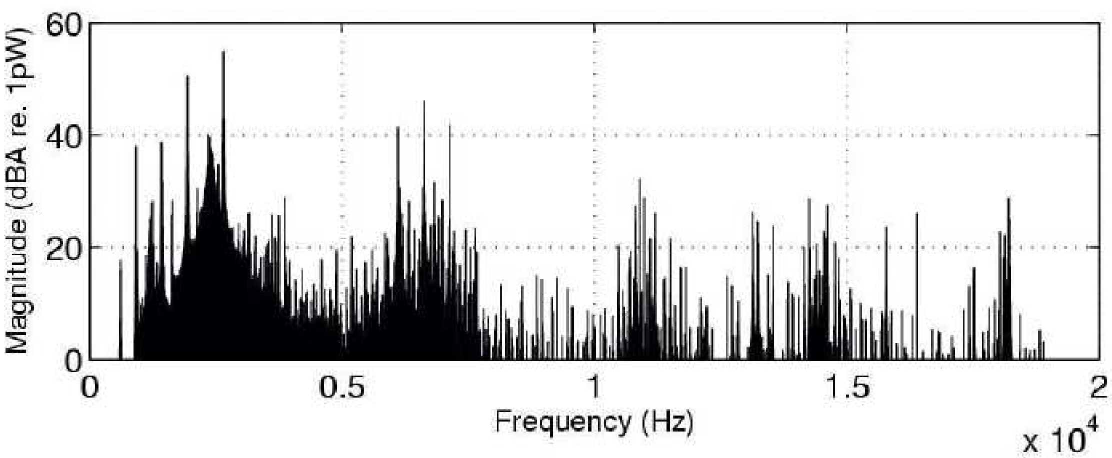
$$L_w = 10 \log_{10}\Big(\sum_{f,m} W_m(f)/W_0\Big), \quad W_0 = 10^{-12} W$$

II Simulation results

-700 W 3-phase induction machine Z_s=27, Z_r=21, p=2, fractional-slot winding, in sinusoidal case (n=1, m=1, s=3%)



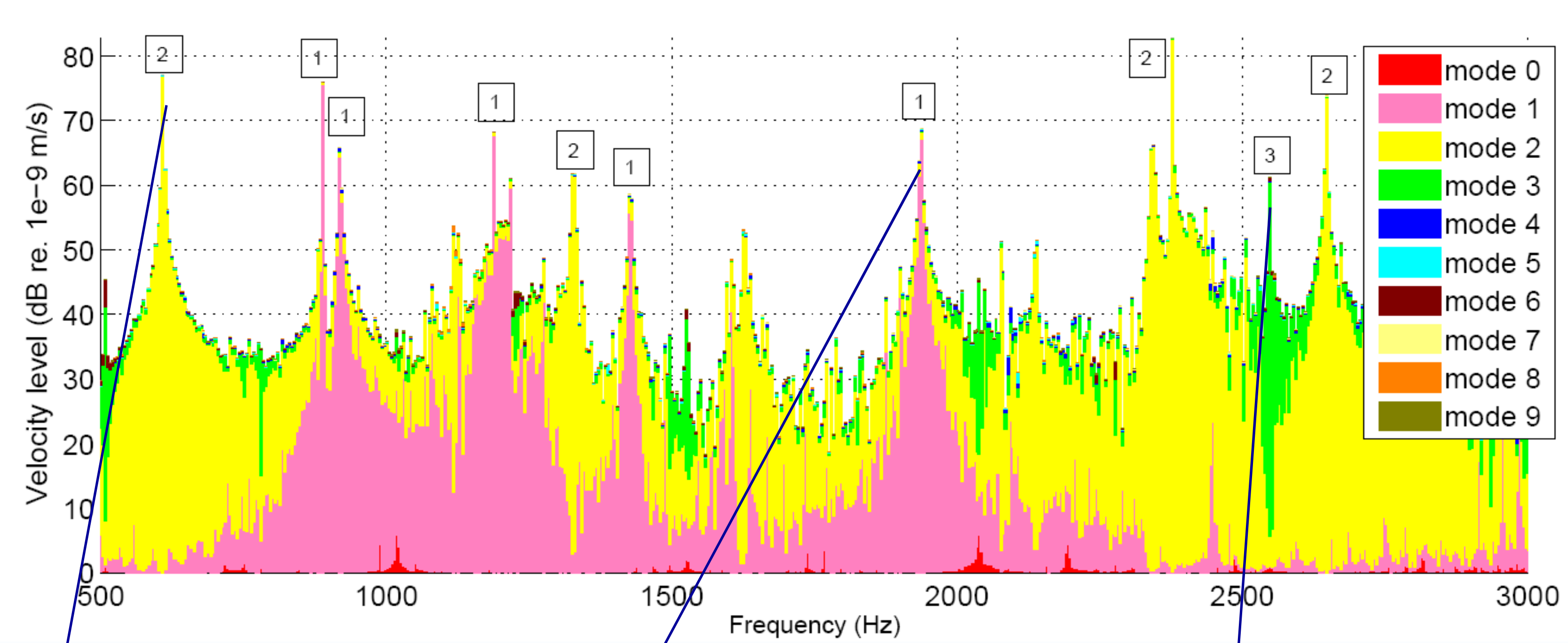
Rotor and stator mmf,
(s=3%, f_1 =50 Hz)



Noise spectrum
(s=3%, f_1 =50 Hz)

II Simulation results

Vibration spectrum with modal contribution (s=3%, f_1 =50 Hz)



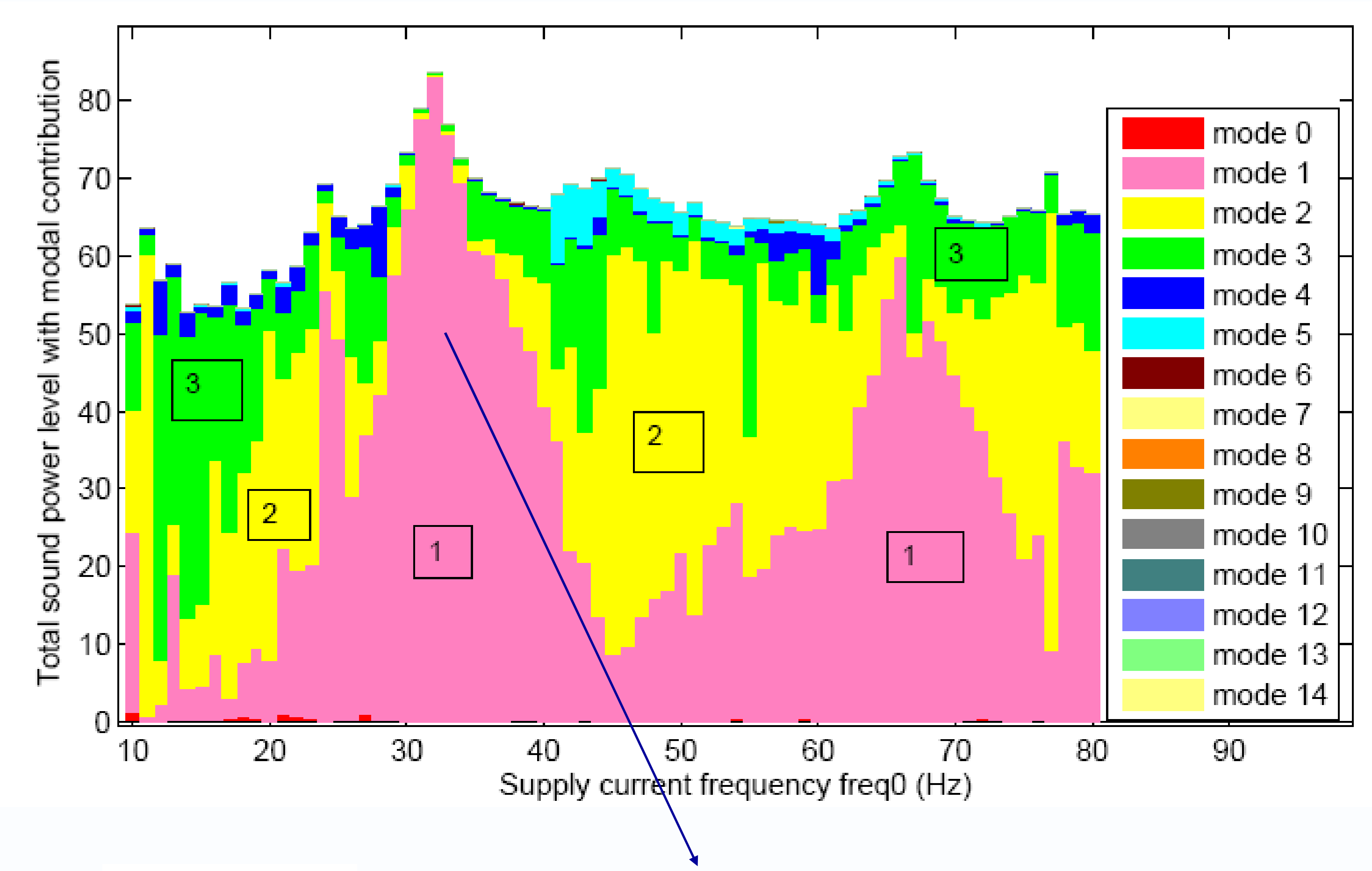
M=2
F=610 Hz $Z_s - Z_r - 2p$ $f_1((1-s)\frac{Z_r}{p}+2)$

M=1
F=1935 Hz $3Z_s - 4Z_r + 2p$ $f_1(4(1-s)\frac{Z_r}{p}-2)$

M=3
F=2550 Hz $4Z_s - 5Z_r$ $f_1(5(1-s)\frac{Z_r}{p})$

II Simulation results

Total noise level with modal contribution (<u>variable speed</u>)



$3Z_s - 4Z_r + 2p$

$f_1(4(1-s)\frac{Z_r}{p} - 2)$

line in resonance with *m*=1 natural frequency of 1100 Hz

III Analytical noise spectrum

- **noise spectrum (= vibration spectrum) is a complex combination of time and space harmonics**

- **to obtain this spectrum, all the functions (currents, turns functions, permeance) are 2D-Fourier expanded**

This way, we obtain stator and rotor mmf spectra:

$$f_R = f_1(1-s)/p$$

Name/Amplitude	Spatial modes	Frequencies	Comments
F_s	$\nu_s = p + \epsilon_s q_s h_s$	$\epsilon_s f_n^s$	$h_s \geq 0$
F_r	$\nu_r = h_r Z_r + \epsilon_r \nu_s$	$\nu_r f_R + \epsilon_r f_{\nu_s n}^r$	$h_r \geq 0$

Stator and rotor mmf waves

Stator current frequencies

Fractional slot space harmonics, otherwise $p \pm 2p\, q_s\, h_s = 5p, 7p, 11p, 13p$ …

±1 factor to take into account propagation direction

Rotor current frequencies

III Analytical noise spectrum

permeance spectrum:

Rotor and stator slotting permeance waves →

Name/Amplitude	Spatial modes	Frequencies	Comments
P_0	0	0	
P_s	$k_s Z_s$	0	$k_r, k_s \geq 1$
P_r	$k_r Z_r$	$-k_r Z_r f_R$	
P_{sr}	$k_s Z_s + \eta k_r Z_r$	$-\eta k_r Z_r f_R$	

air-gap flux density spectrum:

Combination of permeance waves and mmf waves →

Name/Amplitude	Spatial modes	Frequencies
$P_0 F_s$	$\eta_{0s} \nu_s$	$\eta_{0s} \epsilon_s f_n^s$
$P_0 F_r$	$\eta_{0r} \nu_r$	$\eta_{0r} (\nu_r f_R + \epsilon_r f_{\nu_s n}^r)$
$P_s F_s$	$k_s Z_s + \eta_{ss} \nu_s$	$\eta_{ss} \epsilon_s f_n^s$
$P_s F_r$	$k_s Z_s + \eta_{sr} \nu_r$	$\eta_{sr} (\nu_r f_R + \epsilon_r f_{\nu_s n}^r)$
$P_r F_s$	$k_r Z_r + \eta_{rs} \nu_s$	$-k_r Z_r f_R + \eta_{rs} \epsilon_s f_n^s$
$P_r F_r$	$k_r Z_r + \eta_{rr} \nu_r$	$-k_r Z_r f_R + \eta_{rr} (\nu_r f_R + \epsilon_r f_{\nu_s n}^r)$

and finally force spectrum (eliminating high modes, low frequencies and redundant lines):

Name/Amplitude	Spatial modes	Frequencies
$L_1 = P_s F_s P_s F_r$	$\eta_{ss} \nu_s - \eta_{sr} \nu_r$	$\eta_{ss} \epsilon_s f_{n_1}^s - \eta_{sr} (\nu_r f_R + \epsilon_r f_{\nu_s n_2}^r)$
$L_2 = P_s F_s P_r F_s$	$k_s Z_s - k_r Z_r + \nu_s \eta_{ss} - \nu'_s \eta_{rs}$	$k_r Z_r f_R + f_{n_1}^s \epsilon_s \eta_{ss} - f_{n_2}^s \epsilon'_s \eta_{rs}$
$L_3 = P_s F_s P_r F_r$	$k_s Z_s - k_r Z_r + \eta_{ss} \nu_s - \eta_{rr} \nu_r$	$(k_r Z_r - \eta_{rr} \nu_r) f_R + \epsilon_s \eta_{ss} f_{n_1}^s - \epsilon_r \eta_{rr} f_{\nu_s n_2}^r$
$L_4 = P_s F_r P_r F_r$	$k_s Z_s - k_r Z_r + \eta_{sr} \nu'_r - \eta_{rr} \nu_r$	$f_R (k_r Z_r + \eta_{sr} \nu'_r - \eta_{rr} \nu_r) + \epsilon_r \eta_{sr} f_{\nu_s n_1}^r - \epsilon'_r \eta_{rr} f_{\nu_s n_2}^r$
$L_5 = P_r F_s P_r F_r$	$\eta_{rs} \nu_s - \eta_{rr} \nu_r$	$\eta_{rs} \epsilon_s f_{n_1}^s - \eta_{rr} (\nu_r f_R + \epsilon_r f_{\nu_s n_2}^r)$
$L_6 = P_0 F_s P_0 F_s$	$\eta_{0s} \nu_s - \eta'_{0s} \nu'_s$	$\epsilon_s \eta_{0s} f_{n_1}^s - \epsilon'_s \eta'_{0s} f_{n_2}^s$
$L_7 = P_0 F_r P_0 F_r$	$\eta_{0r} \nu_r - \eta'_{0r} \nu'_r$	$f_R (\eta_{0r} \nu_r - \eta'_{0r} \nu'_r) + \epsilon_r \eta_{0r} f_{\nu_s n_1}^r - \epsilon'_r \eta'_{0r} f_{\nu_s n_2}^r$
$L_8 = P_0 F_r P_0 F_s$	$\eta_{0r} \nu_r - \eta_{0s} \nu_s$	$\eta_{0r} (\nu_r f_R + \epsilon_r f_{\nu_s n_1}^r) - \epsilon_s \eta_{0s} f_{n_2}^s$

III Analytical noise spectrum

<u>EXAMPLE</u> (sinusoidal case):

stator mmf wave $F_s=(p,f_1)$
+
stator slotting permeance wave $P_s=(k_sZ_s,0)$

→ flux density wave $P_sF_s=(p+k_sZ_s, f_1+0)$

+

stator mmf wave $F_s=(p,f_1)$
+
rotor slotting permeance wave $P_r= (k_rZ_r,-k_rZ_rf_R)$

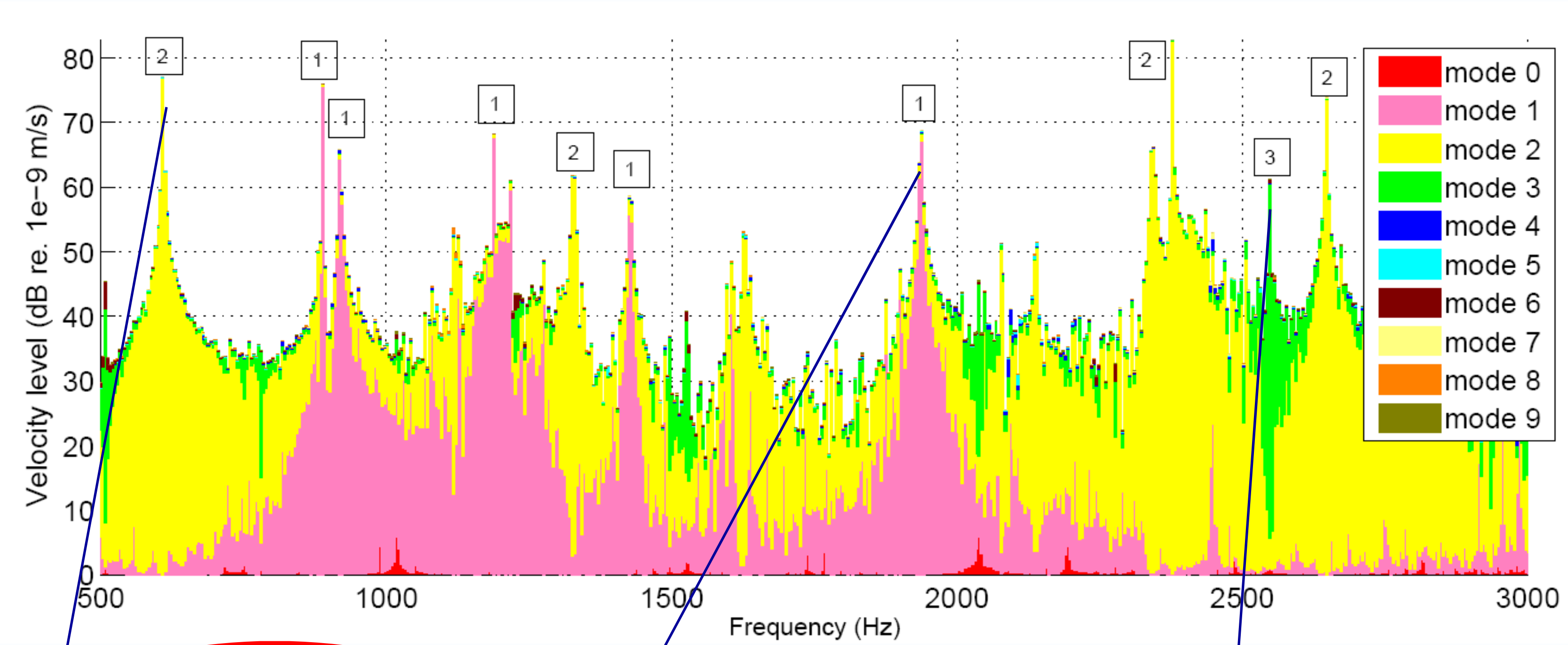
→ flux density wave $P_rF_s=(p-k_rZ_r, f_1+k_rZ_rf_R)$

force wave
of mode $2p+ k_sZ_s- k_rZ_r$
and frequency $f_1(k_rZ_r (1-s)/p+2)$

Name/Amplitude	Spatial modes	Frequencies
$L_1 = P_sF_sP_sF_r$	$\eta_{ss}\nu_s - \eta_{sr}\nu_r$	$\eta_{ss}\epsilon_s f^s_{n_1} - \eta_{sr}(\nu_r f_R + \epsilon_r f^r_{\nu_s n_2})$
$L_2 = P_sF_sP_rF_s$	$k_sZ_s - k_rZ_r + \nu_s\eta_{ss} - \nu'_s\eta_{rs}$	$k_rZ_rf_R + f^s_{n_1}\epsilon_s\eta_{ss} - f^s_{n_2}\epsilon'_s\eta_{rs}$
$L_3 = P_sF_sP_rF_r$	$k_sZ_s - k_rZ_r + \eta_{ss}\nu_s - \eta_{rr}\nu_r$	$(k_rZ_r - \eta_{rr}\nu_r)f_R + \epsilon_s\eta_{ss}f^s_{n_1} - \epsilon_r\eta_{rr}f^r_{\nu_s n_2}$
$L_4 = P_sF_rP_rF_r$	$k_sZ_s - k_rZ_r + \eta_{sr}\nu'_r - \eta_{rr}\nu_r$	$f_R(k_rZ_r + \eta_{sr}\nu'_r - \eta_{rr}\nu_r) + \epsilon_r\eta_{sr}f^r_{\nu_s n_1} - \epsilon'_r\eta_{rr}f^r_{\nu_s n_2}$
$L_5 = P_rF_sP_rF_r$	$\eta_{rs}\nu_s - \eta_{rr}\nu_r$	$\eta_{rs}\epsilon_s f^s_{n_1} - \eta_{rr}(\nu_r f_R + \epsilon_r f^r_{\nu_s n_2})$
$L_6 = P_0F_sP_0F_s$	$\eta_{0s}\nu_s - \eta'_{0s}\nu'_s$	$\epsilon_s\eta_{0s}f^s_{n_1} - \epsilon'_s\eta'_{0s}f^s_{n_2}$
$L_7 = P_0F_rP_0F_r$	$\eta_{0r}\nu_r - \eta'_{0r}\nu'_r$	$f_R(\eta_{0r}\nu_r - \eta'_{0r}\nu'_r) + \epsilon_r\eta_{0r}f^r_{\nu_s n_1} - \epsilon'_r\eta'_{0r}f^r_{\nu_s n_2}$
$L_8 = P_0F_rP_0F_s$	$\eta_{0r}\nu_r - \eta_{0s}\nu_s$	$\eta_{0r}(\nu_r f_R + \epsilon_r f^r_{\nu_s n_1}) - \epsilon_s\eta_{0s}f^s_{n_2}$

II Simulation results

Vibration spectrum with modal contribution (s=3%, f_1 =50 Hz)



M=2 $Z_s - Z_r - 2p$

F=610 Hz $f_1((1-s)\frac{Z_r}{p}+2)$

L_2

M=1 $3Z_s - 4Z_r + 2p$

F=1935 Hz $f_1(4(1-s)\frac{Z_r}{p}-2)$

M=3 $4Z_s - 5Z_r$

F=2550 Hz $f_1(5(1-s)\frac{Z_r}{p})$

III Analytical noise spectrum

CONSEQUENCES:

In sinusoidal case, we can show that the main lines are:

$$F = f_1((1-s)\frac{k_r Z_r}{p} \pm |_0^2) \qquad M = \pm k_s Z_s \mp k_r Z_r \pm |_0^{2p}$$

In PWM case, besides these lines, the main lines are usually:

$$F = |\pm f_1 \pm f_n^s| \qquad M = 0 \quad \text{or} \quad 2p$$

Name/Amplitude	Spatial modes	Frequencies
$L_1 = P_s F_s P_s F_r$	$\eta_{ss}\nu_s - \eta_{sr}\nu_r$	$\eta_{ss}\epsilon_s f_{n_1}^s - \eta_{sr}(\nu_r f_R + \epsilon_r f_{\nu_s n_2}^r)$
$L_2 = P_s F_s P_r F_s$	$k_s Z_s - k_r Z_r + \nu_s \eta_{ss} - \nu_s' \eta_{rs}$	$k_r Z_r f_R + f_{n_1}^s \epsilon_s \eta_{ss} - f_{n_2}^s \epsilon_s' \eta_{rs}$
$L_3 = P_s F_s P_r F_r$	$k_s Z_s - k_r Z_r + \eta_{ss}\nu_s - \eta_{rr}\nu_r$	$(k_r Z_r - \eta_{rr}\nu_r) f_R + \epsilon_s \eta_{ss} f_{n_1}^s - \epsilon_r \eta_{rr} f_{\nu_s n_2}^r$
$L_4 = P_s F_r P_r F_r$	$k_s Z_s - k_r Z_r + \eta_{sr}\nu_r' - \eta_{rr}\nu_r$	$f_R(k_r Z_r + \eta_{sr}\nu_r' - \eta_{rr}\nu_r) + \epsilon_r \eta_{sr} f_{\nu_s n_1}^r - \epsilon_r' \eta_{rr} f_{\nu_s n_2}^r$
$L_5 = P_r F_s P_r F_r$	$\eta_{rs}\nu_s - \eta_{rr}\nu_r$	$\eta_{rs}\epsilon_s f_{n_1}^s - \eta_{rr}(\nu_r f_R + \epsilon_r f_{\nu_s n_2}^r)$
$L_6 = P_0 F_s P_0 F_s$	$\eta_{0s}\nu_s - \eta_{0s}'\nu_s'$	$\epsilon_s \eta_{0s} f_{n_1}^s - \epsilon_s' \eta_{0s}' f_{n_2}^s$
$L_7 = P_0 F_r P_0 F_r$	$\eta_{0r}\nu_r - \eta_{0r}'\nu_r'$	$f_R(\eta_{0r}\nu_r - \eta_{0r}'\nu_r') + \epsilon_r \eta_{0r} f_{\nu_s n_1}^r - \epsilon_r' \eta_{0r}' f_{\nu_s n_2}^r$
$L_8 = P_0 F_r P_0 F_s$	$\eta_{0r}\nu_r - \eta_{0s}\nu_s$	$\eta_{0r}(\nu_r f_R + \epsilon_r f_{\nu_s n_1}^r) - \epsilon_s \eta_{0s} f_{n_2}^s$

Conclusion

- **analytical model predicting the electromagnetic noise level of a PWM controlled induction machine**

- **method to analytically derive main noise spectrum lines, including the influence of both time and space harmonics**

- **better understanding of harmonic interactions role in magnetic noise generation**

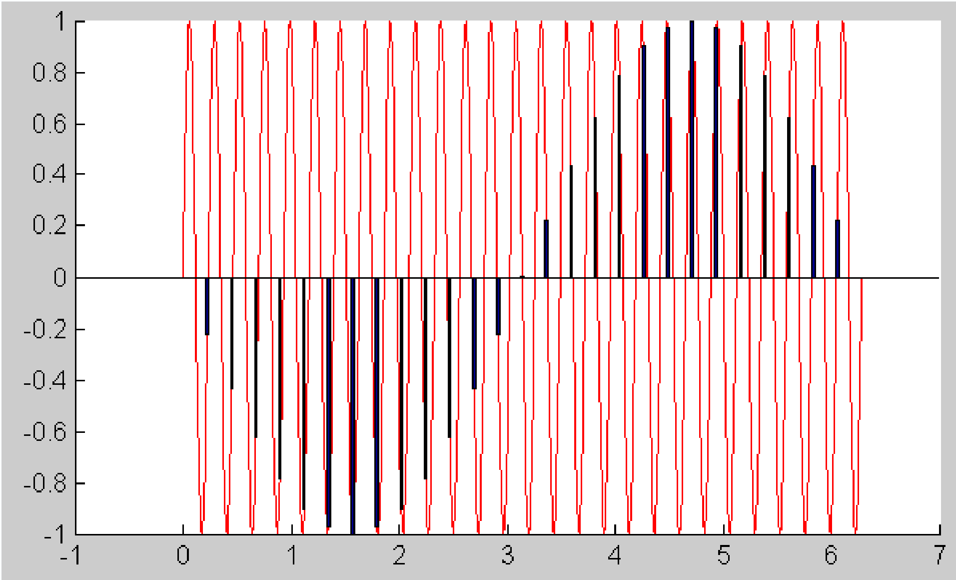
Future work

- **comparison with experiments in PWM case, influence of rotor space harmonics**

- **3D analytical mechanical and acoustic model**

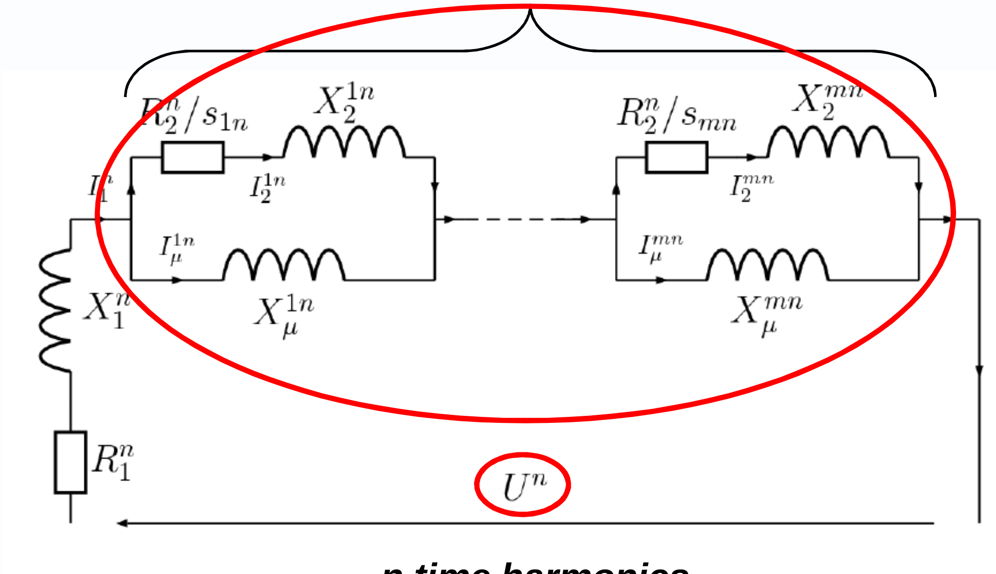
Thank you for your attention.

Spatial aliasing: Z_r=28, m=27



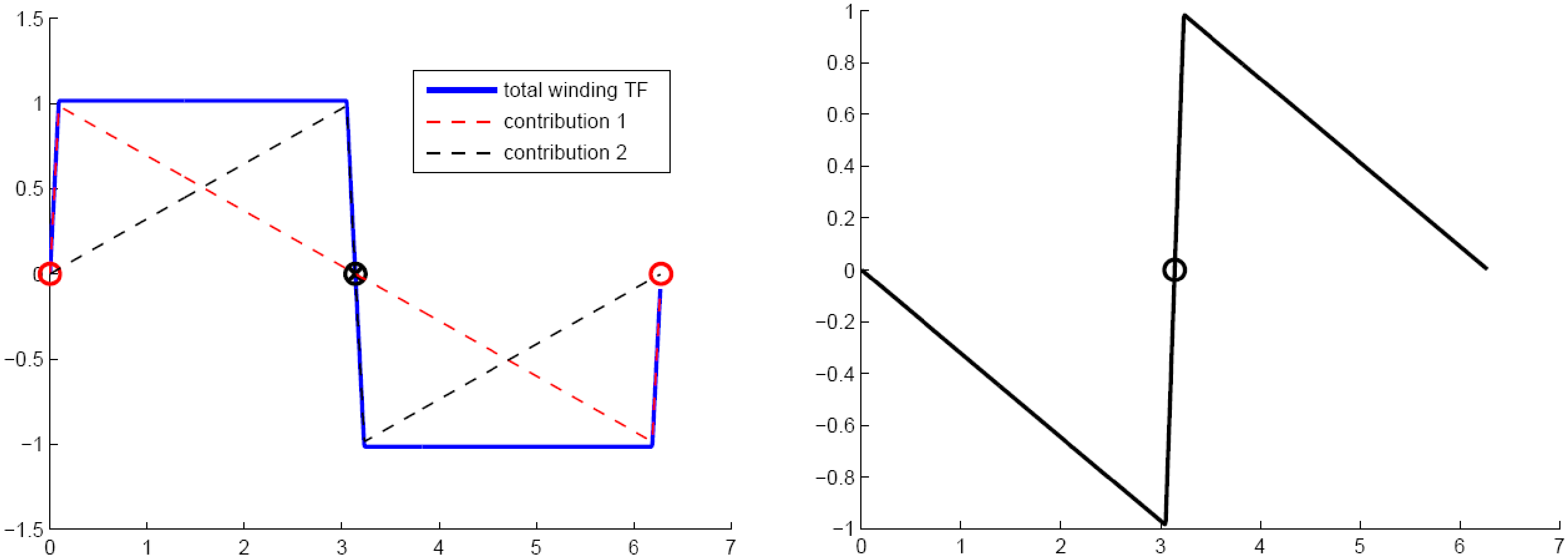
-> space harmonics m>Z_r/2 induced in rotor currents by stator windings must be corrected…

m space harmonics

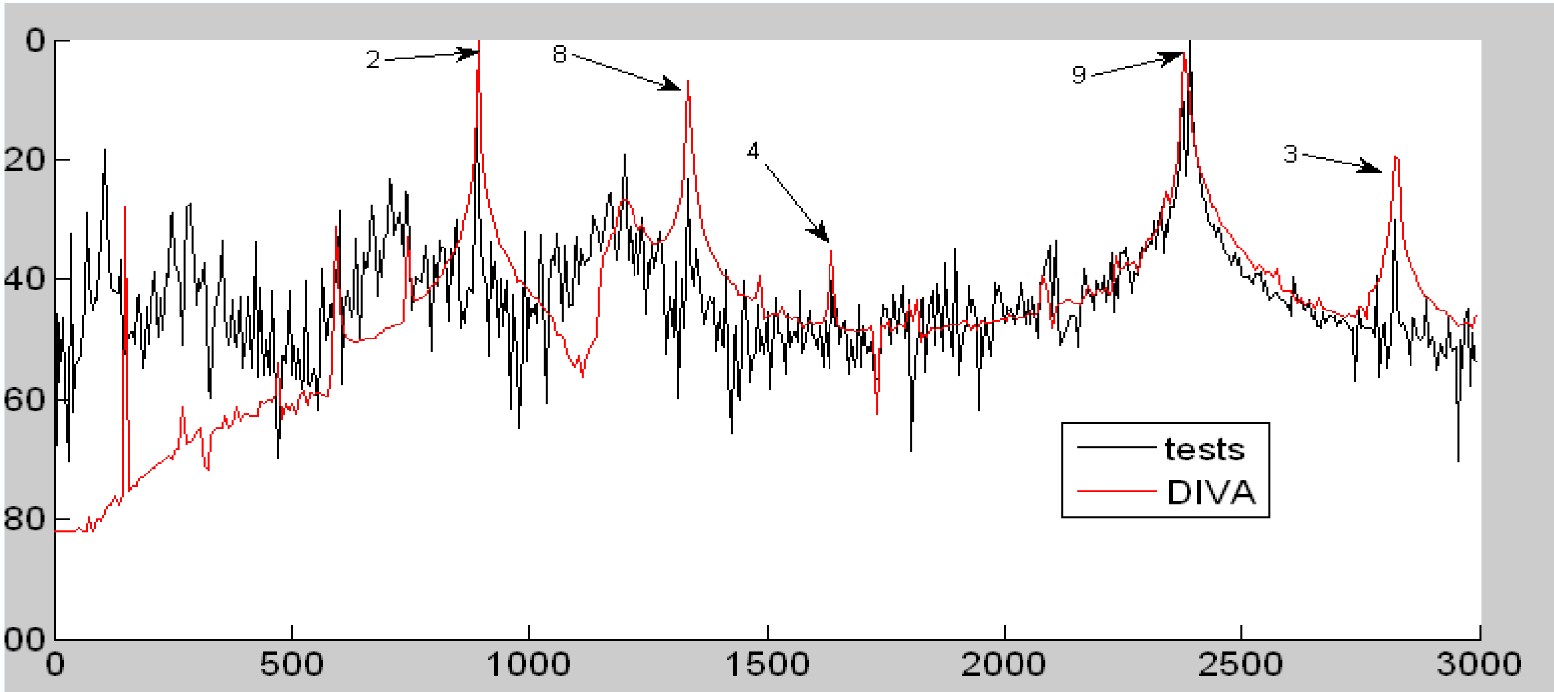


n time harmonics

R_1^n	stator/rotor resistance (*n :* skin effect)
$X_1^n = w_n L_1$	stator reactance
$X_2^n = w_n L_2^m$	rotor reactance (*m :* winding factor in transformation factor)
$X_\mu^n = w_n L_\mu^m$	magnetizing reactance (*m :* winding factor in transformation factor)
s_{mn}	harmonic slip



total winding TF
contribution 1
contribution 2
1.5
1
0.5
0
−0.5
−1
−1.5
0
1
2
3
4
5
6
7
0.8
0.6
0.4
0.2
−0.2
−0.4
−0.6
−0.8



Spectre vibratoire mesuré/simulé, g=5.6% f=75 Hz

$$f_m = f_0 \Gamma \frac{m(m^2-1)}{\sqrt{m^2+1}} \qquad \Gamma = \frac{h_c}{2\sqrt{3}R_c}$$

$$F(t, \alpha_s) = \sum A_{mn} \sin(m\alpha_s + 2\pi f_n t + \phi_{mn}) = \sum A_{mn} \{(m, f_n)\} = \{(m_i, f_i)\}_{i \in I}$$

the following rules are used:

$$\{(m_i, f_i)\}_{i \in I} + \{(m_j, f_j)\}_{j \in J} = \{(m_i, f_i)\}_{i \in I \cup J}$$

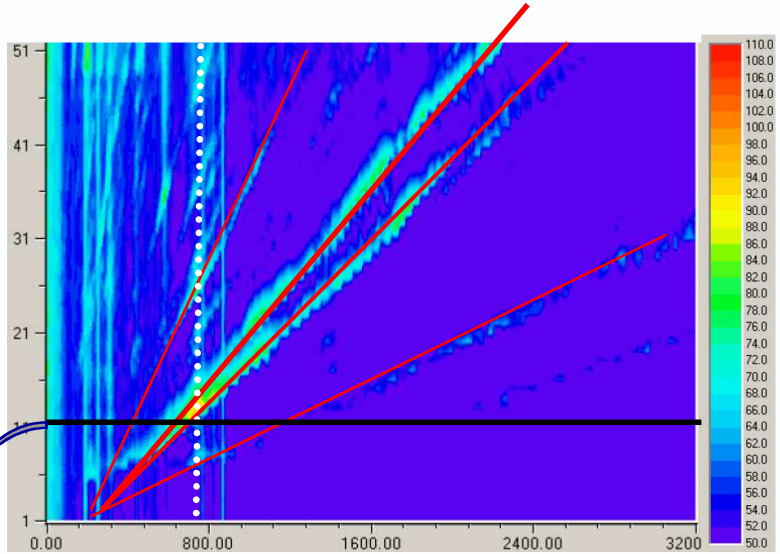
$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\{(m_i, f_i)\}_{i \in I} \times \{(m_j, f_j)\}_{j \in J} = \{(m_i + \eta_{ij} m_j, f_i + \eta_{ij} f_j)\}_{i \in I, j \in J, \eta_{ij} = \pm 1}$$

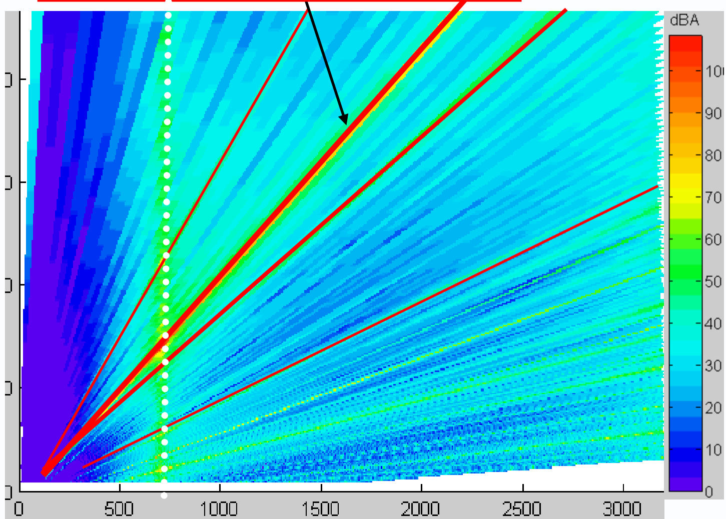
Zs=36, Zr=28 - 250kW
shorted-pitch winding

Slotting magnetic line
fs(Zr/p+2)

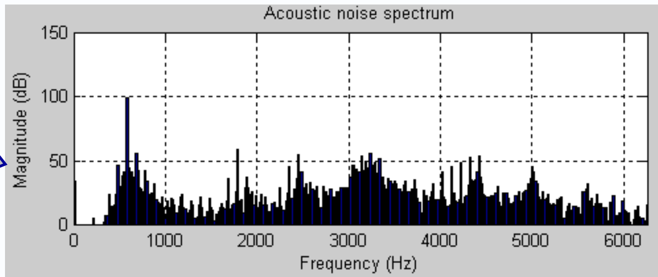
mode 2=Zs-Zr-2p



Mesures (dBA)
Bruit magnétique+mécanique



Simulation DIVA (dBA)
Bruit magnétique seul



I Analytical model: electromagnetic part

$$B(t,\alpha^s) = \Lambda(t,\alpha^s)\left(f_{mm}^r(t,\alpha^s) + f_{mm}^s(t,\alpha^s)\right)$$

Evolution of **permeance***, stator mmf, rotor mmf and air-gap radial flux density in function of time*