Application Limits of the Airgap Maxwell Tensor

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***Abstract*—In an electrical machine, the Maxwell Tensor is widely used to compute global forces or local pressure along a surface in the air. This communication proposes to highlight the limits of the method with an academic case of slotless stator and rotor. In particular an analytic demonstration shows the existence of coefficients depending on the geometry and the wavenumber between the application of the Maxwell Tensor in the air-gap and the stator magnetic pressure.**

***Index Terms*—Maxwell Tensor, Magnetic pressure, Electrical machines, Magneto-mechanical, Vibration.**

I. Introduction

The Maxwell Tensor (MT) is widely used to compute local surface pressure and integrated forces on a given surface surrounding a body. Nonetheless the location of this surface in the studied domain has an important impact on the results [1]. Then this communication proposes to highlight the limits of the application of MT in the air-gap. It is performed by comparison with the theoretic magnetic pressure which is exactly the application of the MT at the air-ferromagnetic interface for linear isotropic media [2]. The deviation between the applications of Maxwell Tensor are quantified with coefficients depending on the geometry and the wavenumber.

II. Methodology

Understanding the sources of MT pressure variations is a difficult task because of numerous artifacts that can be produced by the numerical simulation of electrical machines such as slotting effect, sharp geometries, interference between the wave-numbers, *etc*. To avoid these artifacts, an academic slotless machine in Fig.1 (left) is studied. In order to have only one magnetic wave-number, the magnetic potential z-component A_z is imposed at radius R_ag such that $\forall\theta \in [0, 2\pi]$, $A_\mathrm{z}(R_\mathrm{ag}, \theta) = \beta \sin(n\theta)$, $n \in \mathbf{N}^*$ and on the stator external yoke R_e as $A_\mathrm{z}(\mathrm{R_e}, \theta) = 0$. Then the 2D linear Poisson equation can be solved analytically for A_z according to [3]. Thus the magnetic potential, flux and field can be analytically computed in the studied domain. Assuming an infinite permeability in the ferromagnetic media leads to (1) for the only non-zero magnetic potential component:

$$A_\mathrm{z}(r,\theta) = \beta \left(E_n(r, \mathrm{R_s}) + \frac{2E_n(\mathrm{R_{ag}}, r)}{F_n(\mathrm{R_{ag}}, \mathrm{R_s})} \right) \frac{\sin(n\theta)}{E_n(\mathrm{R_{ag}}, \mathrm{R_s})} \quad (1)$$

with $E_n : (x, y) \to \frac{x^n}{y^n} - \frac{y^n}{x^n}$, and $F_n : (x, y) \to \frac{x^n}{y^n} + \frac{y^n}{x^n}$ two polynomial functions. Now the MT magnetic pressure $P(r, \theta) = \frac{1}{2\mu_0}\left(B_r^2(r, \theta) - B_\theta^2(r, \theta)\right)$ can be analytically computed, with μ_0 considered as the air permeability. Performing a Fourier decomposition, the MT magnetic pressure can be decomposed as follows:

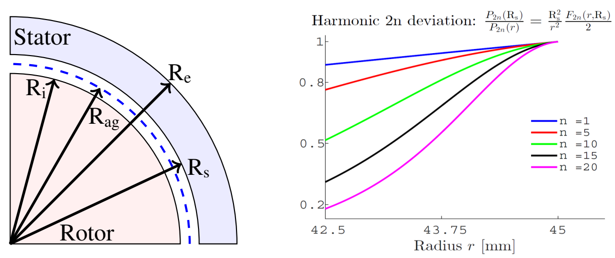

Fig. 1: (Left) Slot-less electrical machine used to compare magnetic pressure on the stator and in the air-gap: $\mathrm{R_s} = 45\,\mathrm{mm}$ and $\mathrm{R_{ag}} = 42.5\,\mathrm{mm}$. (Right) Comparing stator and air-gap Maxwell Tensor magnetic pressure

$$P(r,\theta) : \begin{cases} r \in [\mathrm{R_{ag}}, \mathrm{R_s}] \\ \theta \in [0, 2\pi] \end{cases} \to P_0(r) + P_{2n}(r)\cos(2n\theta) \quad (2)$$

Although commonly used at any value of $r \in [\mathrm{R_{ag}}, \mathrm{R_s}]$, this result is only demonstrated for $r = \mathrm{R_s}$ [2]. Using (1) to compute analytically (2) leads to (3) which allows to compare the airgap MT and the theoretic magnetic pressure.

$$P(r,\theta) = \frac{\mathrm{R_s^2}}{r^2}\left(P_0(\mathrm{R_s}) + P_{2n}(\mathrm{R_s})\frac{F_{2n}(r, \mathrm{R_s})}{2}\cos(2n\theta) \right) \quad (3)$$

The analytic differences between theoretic pressure and airgap pressure lie in the two coefficients $\frac{\mathrm{R_s^2}}{r^2}$ and $\frac{F_{2n}(r,\mathrm{R_s})}{2}$. Note that it does not depend on θ and β. These two coefficient show that air-gap MT results depend on the radius of application such that it can affects magnetic pressure harmonics calculation. The impact on the 2n harmonic of the MT pressure is presented in Fig.1 (right). In the vibro-acousic study of electrical machines, such phenomenon can have an important impact on high wave-number's amplitude, such that the MT applied at the air-stator interface should be the first choice. However the application of the MT at the interface is source of numerical errors [1] such that the application of the MT in the air-gap is of great interest. In the extended paper, the demonstration of (3) and the physical explanations will be provided.

References

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