Application Limits of the Airgap Maxwell Tensor

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OBJECTIVES

- Compare airgap application of the Maxwell stress tensor with analytic magnetic stress.
- Propose analytic coefficient to transfer Maxwell stress from airgap to stator or rotor bore radius.
- Define an analytic case to be used as reference for electromagnetic noise and vibration studies.

MAXWELL TENSOR

The magnetic stress tensor for incompressible body with linear magnetization - $\boldsymbol{B} = \mu\, \boldsymbol{H}$ - reduces to [1]:

$$\boldsymbol{T}_{\mathbf{m}} = -\frac{\mu}{2}\,(\boldsymbol{H}\cdot\boldsymbol{H})\,\mathbf{I} + \mu\,\boldsymbol{H}\,\boldsymbol{H} \quad (1)$$

- Applying divergence of (1) between air and ferromagnetic media leads to the following magnetic stress [1,2]:

$$\boldsymbol{P}^{\boldsymbol{S}} = \left(\frac{1}{2}\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)B_{\mathrm{n}}^2 - \frac{\mu_0 - \mu}{2}H_{\mathrm{t}}^2\right)\mathbf{n} \quad (2)$$

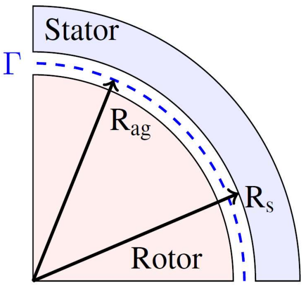
- Applying Stoke's theorem on (1) along a circular contour in the middle of the airgap leads to the following approximation of the magnetic radial stress [3]:

$$\boldsymbol{P}_{\mathbf{r}}^{\mathbf{A}} = \left(\frac{1}{2\mu_0}B_{\mathrm{r}}^2 - \frac{\mu_0}{2}\,H_{\theta}^2\right)\boldsymbol{e}_{\mathbf{r}} \quad (3)$$

SINGLE MAGNETIC WAVE

In order to compare (2) and (3), an **academic slotless machine** is studied:

- **Slotless stator** and rotor to avoid sharp geometries and slotting effect. Infinite permeability is assumed.
- **Analytical solving** $\forall \boldsymbol{r} \in [\mathrm{R_{ag}}, \mathrm{R_s}]$, $\forall\theta \in [0, 2\pi]$ of magnetic potential vector $A_z(r,\theta)$ to avoid meshing and numerical errors.
- **Single-wave excitation** to avoid interference between different wavenumbers with a boundary condition at $\mathrm{R_{ag}}$:

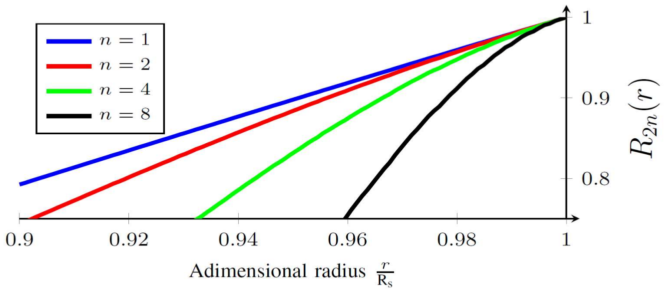


$$A_z(\mathrm{R_{ag}},\theta) = \beta\,\sin(n\theta + \phi)$$

An analytical magnetic radial **transfered** stress $\boldsymbol{P}_{\boldsymbol{r}}^{\boldsymbol{P}}$ exists between $\mathrm{R_{ag}}$ and $\mathrm{R_s}$ such that:

$$\boldsymbol{P}_{\mathbf{r}}^{\mathbf{P}}(\theta) = \frac{P_{\mathrm{r},0}^{\mathrm{A}}}{\mathrm{R}_0} + \frac{P_{\mathrm{r},2\mathrm{n}}^{\mathrm{A}}}{\mathrm{R}_{2\mathrm{n}}}\cos(2\mathrm{n}\,\theta + 2\phi) = \boldsymbol{P}^{\boldsymbol{S}}(\theta)$$

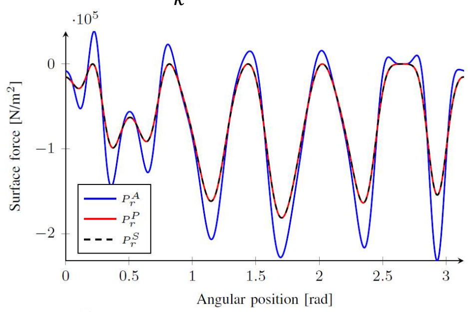
With $\mathbf{R}_{\boldsymbol{k}} = \frac{1}{2}\left(\frac{\mathrm{R_{ag}}}{\mathrm{R_s}}\right)^{k-2} + \frac{1}{2}\left(\frac{\mathrm{R_s}}{\mathrm{R_{ag}}}\right)^{k+2}$ a **transfer** coefficient.

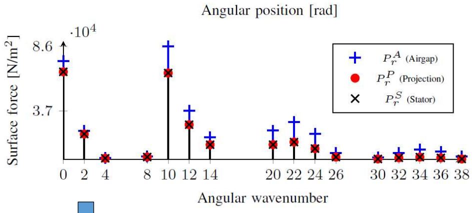


$$\boldsymbol{P}^{\boldsymbol{S}} \approx \boldsymbol{P}_{\boldsymbol{r}}^{\boldsymbol{A}} \quad \textbf{if} \quad \mathbf{R}_{\boldsymbol{k}} \approx 1 \;\leftrightarrow\; k\,\frac{\mathrm{R_s}-\mathrm{R_{ag}}}{\mathrm{R_{ag}}} \ll 1$$

MULTI-HARMONIC GENERALIZATION

$$\boldsymbol{P}_{\boldsymbol{r}}^{\mathbf{P}}(\theta) = \sum_k \frac{P_{\mathrm{r},k}^{\mathrm{A}}}{\mathrm{R}_k}\cos(2k\,\theta + 2\phi_k)\,\boldsymbol{e}_{\mathbf{r}}$$

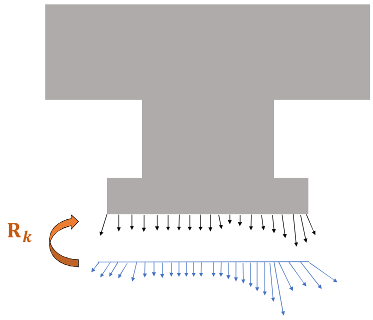




$$\boldsymbol{P}_{\mathbf{r}}^{\mathbf{P}} = \boldsymbol{P}^{\boldsymbol{S}} \neq \boldsymbol{P}_{\boldsymbol{r}}^{\boldsymbol{A}}$$

CONCLUSION

- New transfer law from airgap to stator bore radius has been found.
- It can be applied in front of stator tooth tip.



- Further work has to be achieved to generalize transfer coefficient for slotted area and for tangential magnetic stress.

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