Study of the combined effects of the airgap transfer for Maxwell Tensor and the tooth mechanical modulation in electrical machines

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The Maxwell Tensor (MT) method is widely used to compute global forces or local surface forces for vibroacoustic design of electrical machines under electromagnetic excitation. In particular the airgap Maxwell Tensor method is based on a cylindrical shell in the middle of the airgap. This communication proposes to quantify the differences between the airgap MT and the magnetic force wave experienced by the stator. In particular the airgap to stator transfer and the tooth mechanical modulation effect are studied. A numerical application is performed with a turbo-alternator to illustrate the respective and combined effects of both phenomena. The communication highlights that the tooth mechanical modulation alone is not necessary relevant for electrical machines with a high number of tooth. However the combination of both phenomena has a clear impact on the computed magnetic surface force.

***Index Terms*—Magnetic forces, electric machines, electromagnetics, magnetomechanical effects, vibrations.**

I. Introduction

The Maxwell Tensor (MT) is widely used to compute the magnetic forces that apply to electrical machines subjected to electromagnetic excitation. In particular the airgap MT method is based on a cylindrical surface in the middle of the airgap. Historically the airgap MT is used by electrical machine designers to compute accurately the global electromagnetic torque waveform applying to the machines.

This technique has been extended to the study of local magnetic forces experienced by the outer structure (generally the stator) for vibro-acoustic studies [1]. In order to improve the MT accuracy, recent work [2] suggests the use of transfer coefficients which allow to compute the surface forces at the stator bore radius based on the airgap MT. Although the airgap MT is a good approximation of magnetic force for small airgaps, its application to topologies such as turbo-alternators with relatively wide airgap could result in significant errors for the magnetic force calculations.

Moreover, the shape and position of teeth in electric machines have a discreet dimension. Thus the wave of magnetic force perceived at a tooth head is not the same as that seen by the stator yoke. Indeed recent publications show high spatial wavenumbers on the tooth tip are modulated into low wavenumbers on the stator yoke [3]. This tooth modulation effect alone is considered negligible when the number of teeth is large compared to the studied magnetic force wavenumbers.

This communication proposes to study the application and combination of both phenomena. A numerical application is performed for a turbo-alternator topology.

II. Application of Maxwell Tensor

A. Airgap Maxwell Tensor

According to [4], in an incompressible linearly magnetizable media, the magnetic flux density $\mathbf{B}$ is related to the magnetic field $\mathbf{H}$ $\forall \mathbf{x} \in \Omega$ by $\mathbf{B}(\mathbf{x}) = \mu(\mathbf{x}, B)\mathbf{H}(\mathbf{x})$ such that the magnetic stress tensor reduces to the following form:

$$\mathbf{T}_{\rm m} = -\mu \frac{\mathbf{H}\cdot\mathbf{H}}{2}\mathbf{I} + \mu\mathbf{H}\mathbf{H} \tag{1}$$

with $\mathbf{I}$ the identity tensor. Applying Stoke's theorem along a closed boundary Γ including a volume V gives the total magnetic force $\mathbf{F}_{\rm m}$ acting on this volume. For electrical machines, it is more precise when applied on a circular surface in the middle of the airgap as in Fig. 1 at a radius $\mathrm{R_{ag}}$ [1,5] :

$$\mathbf{F}_{\rm m} = \int_{\rm V} \nabla(\mathbf{T}_{\rm m})\ {\rm dV} = \oint_{\Gamma} -\frac{\mu_0}{2}H^2\mathbf{n} + \mu_0 H_{\rm n}\mathbf{H}\ d\Gamma \tag{2}$$

The airgap MT method assumes the term under the integral to be the magnetic surface force density. Developing this term in the polar referential leads to:

$$\begin{aligned} P_{\rm r}(\mathrm{R_{ag}},\theta) &= \frac{1}{2\mu_0}B_{\rm r}(\mathrm{R_{ag}},\theta)^2 - \frac{\mu_0}{2}H_{\theta}(\mathrm{R_{ag}},\theta)^2 \\ P_{\theta}(\mathrm{R_{ag}},\theta) &= B_{\rm r}(\mathrm{R_{ag}},\theta)H_{\theta}(\mathrm{R_{ag}},\theta) \end{aligned} \tag{3}$$

with $P_{\rm r}$ the radial magnetic surface force density, P_θ the tangential magnetic surface force density, $B_{\rm r}$ the radial magnetic flux density, and H_θ the tangential magnetic field. For the vibroacoustic study of electrical machines, the Fourier transform of the airgap MT is the more useful form:

$$P_{\rm w}(\mathrm{R_{ag}},\theta) = \sum_{n=-\infty}^{n+\infty} \hat{P}_{{\rm w},n}(\mathrm{R_{ag}})\mathrm{e}^{jn\theta} \tag{4}$$

with $\hat{P}_{{\rm w},n}$ the complex amplitude, $\mathrm{w} = \mathrm{r}$ for the radial direction and $\mathrm{w} = \theta$ for the tangential direction. The airgap MT is denoted MT-AG in the following sections.

III. New methods for airgap Maxwell Tensor

The goal of this section is to presents new methods to compute the magnetic force waves experienced by the stator from the airgap MT.

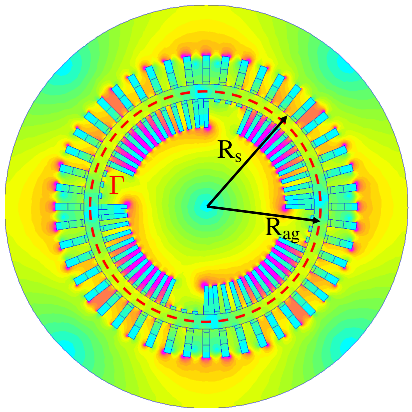

Fig. 1. Turbo-alternator: $Z_s = 48$, $R_{ag} = 0.8$ [m] and $R_s = 0.85$ [m]

A. Transfer of surface forces

In this communication, an extended version of the transfer coefficients found in [2] is proposed. The wavenumber n of airgap MT surface force evolves between the radius of application R_{ag} and the stator bore radius R_s according to the transfer law for the complex Fourier transforms $\hat{P}_{r,n}$ and $\hat{P}_{\theta,n}$:

$$\begin{aligned} \hat{P}_{r,n}(R_s) &= S_n \hat{P}_{r,n}(R_{ag}) + jC_n \hat{P}_{\theta,n}(R_{ag}) \\ \hat{P}_{\theta,n}(R_s) &= S_n \hat{P}_{\theta,n}(R_{ag}) - jC_n \hat{P}_{r,n}(R_{ag}) \\ S_n &= \frac{1}{2}\left(\left(\frac{R_{ag}}{R_s}\right)^{n+2} + \left(\frac{R_{ag}}{R_s}\right)^{-n+2}\right) \\ C_n &= \frac{1}{2}\left(\left(\frac{R_{ag}}{R_s}\right)^{n+2} - \left(\frac{R_{ag}}{R_s}\right)^{-n+2}\right) \end{aligned} \quad (5)$$

where S_n is the self-transfer coefficient and C_n the cross-transfer coefficient. The transformation of MT-AG with these formulae is denoted MT-TR in the following sections. These coefficients are valid for any topology, unlike previous work [2], in particular for wide airgap topologies where the MT-AG highly depends on the radius of application.

B. Tooth Mechanical Modulation

The mechanical behaviour of a slotted stator is different from a slotless one. Considering a stator with Z_s the number of teeth, the surface force $\hat{P}_{r,n}(R_s)$ of wavenumber n such that $|n| > |\frac{Z_s}{2}|$ is modulated into a surface force $\hat{P}_{r,m}(R_s)$ of wavenumber m experienced by the stator yoke such that $m = n - kZ_s$ with $k \in \mathbb{Z}$ and $|m| \leq |\frac{Z_s}{2}|$ [3]. In this communication, the following MT modulation law is proposed to compute the equivalent modulated surface force:

$$\hat{P}_{w,m}(R_s) = \frac{m}{n}\frac{\sin(\frac{n\pi}{Z_s})}{\sin(\frac{m\pi}{Z_s})}\hat{P}_{w,n}(R_s) \quad (6)$$

with $w = r$ for the radial direction and $w = \theta$ for the tangential direction. This transformation is denoted MT-MOD in the following.

IV. Application to a turbo-alternator

The previous section presented two methods MT-TR and MT-MOD which allow to improve the MT-AG accuracy. The successive application of MT-TR then MT-MOD is called MT-TRMOD in the rest of this communication. The Fig. 2 focuses on magnetic surface force density wavenumbers of interest for vibroacoustic in both radial and tangential directions. All the previous methods are compared for the turbo-alternator topology. The magnetic field is solved using Finite Element Analysis.

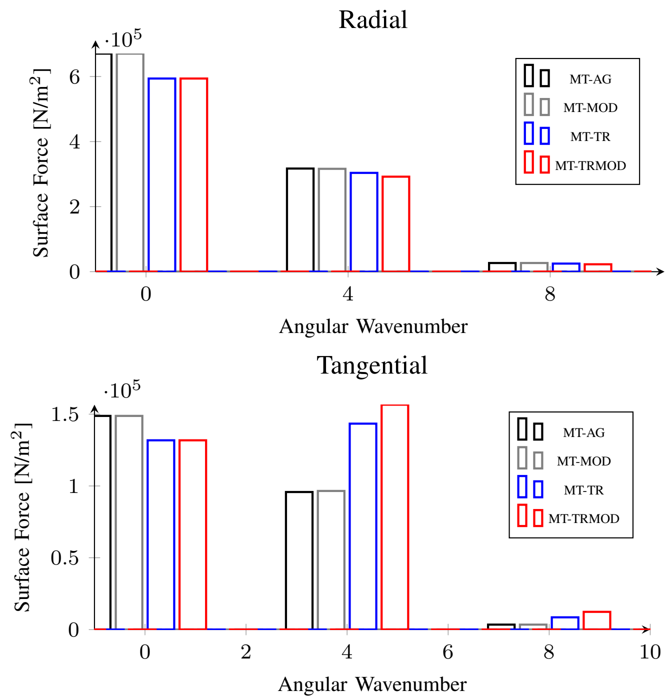

Fig. 2. Comparison of several methods for surface force density: MT-TR is the reference method at the tooth tip, and MT-TRMOD the reference at the yoke level.

It can be observed in Fig. 2 that MT-TR transformation has a non-negligible impact on the surface force wavenumbers, in particular the 4^{th} tangential wavenumber. The tooth mechanical modulation alone MT-MOD has a negligible impact on the surface force spectrum. However, it can be observed with MT-TRMOD that the modulation amplifies the effect of the transfer MT-TR.

Therefore, the combined use of both methods is recommended. In the extended paper, the demonstration of (5) and (6) with the physical explanations, the numerical value used to solve the electromagnetic field in the turbo-alternator and additional discussions about the physical meaning of these surface forces will be provided.

References

[1] M. Valavi, A. Nysveen, R. Nilssen, R. D. Lorenz, and T. Rølvåg, "Influence of pole and slot combinations on magnetic forces and vibration in low-speed PM wind generators," *IEEE Trans. Magn.*, vol. 50, no. 5, pp. 1–11, 2014.

[2] R. Pile, G. Parent, E. Devillers, T. Henneron, Y. Le Menach, J. Le Besnerais, and J.-P. Lecointe, "Application Limits of the Airgap Maxwell Tensor," in *CEFC*, 2018.

[3] H. Fang, D. Li, R. Qu, and P. Yan, "Modulation effect of slotted structure on vibration response in electrical machines," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 4, pp. 2998–3007, April 2019.

[4] R. E. Rosensweig, *Ferrohydrodynamics*. Cambridge University Press, 1985.

[5] Z. Ren, "Comparison of Different Force Calculation Methods in 3D Finite Element Modelling," *IEEE Trans. Magn.*, vol. 30, no. 5, pp. 3471–3474, 1994.